Faculty of Engineering
Cairo University

Geothermal Energy Capacity Building in Egypt (GEB)

Thermal Engineering for Geothermal Energy

Co-funded by the
Erasmus+ Programme
of the European Union

Objectives – Contents – Outcomes (1)

Objectives

- ☐ An introductory overview of thermodynamics
- ☐ Some basic aspects of fluid mechanics
  (flow in pipes, fluid machines)
- ☐ Some basic aspects of heat transfer
  (with a special focus on conduction – heat transfer in soil)

The course will present the fundamentals of these topics in a classical approach, creating a solid base that will allow the students to analyse, in later subjects, energy systems in general and geothermal systems in particular.

Prerequisites

- ☐ An introductory course of graduate physics

Objectives – Contents – Outcomes (2)

Contents

Part I – Thermodynamics

- *Basic concepts*
- *Energy concepts*
- *Pure substance & Mixtures*
- *First Law*
- *Second Law*
- *Entropy*
- *Power cycles*
- *Refrigeration cycles*

Part II – Transport phenomena

- *Fluid properties & basic laws*
- *Losses in pipes*
- *Description of fluid machines*
- *Modes of heat transfer*
- *Conduction basics*
- *Conduction in variable sections & fins*
- *Heat transfer in soil*

Objectives – Contents – Outcomes (2)

Outcomes

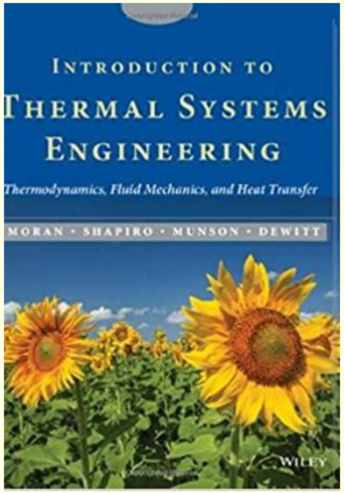
At the end of this course, trainee should be able to:

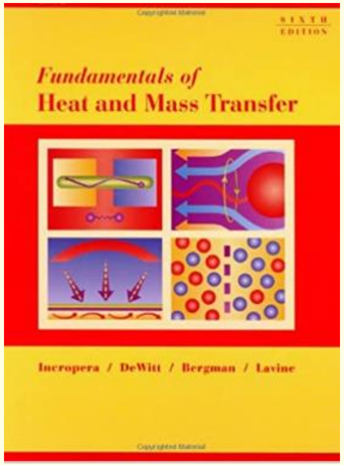
- ***Identify, quantify different energies, and perform energy balances***
- ***Evaluate thermodynamic properties of matter***
- ***Understand implications of Second Law***
- ***Evaluate entropy and understand its charts***
- ***Understand and analyse power and refrigeration cycles***
- ***Evaluate losses in a hydraulic loop***
- ***Select appropriate fluid machine***
- ***Perform simple heat transfer estimations, especially conduction***
- ***Understand heat transfer mechanisms in soil***

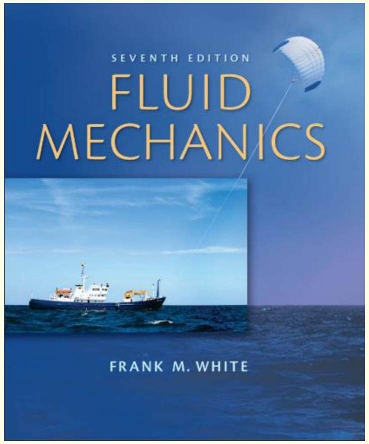
Objectives – Contents – Outcomes (2)

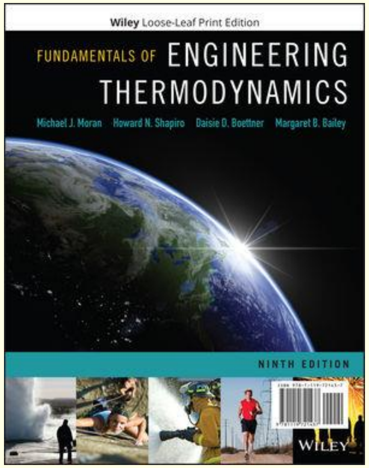
References

- *Introduction to Thermal Systems Engineering: Thermodynamics, Fluid Mechanics, and Heat Transfer. Michael J. Moran, Howard N. Shapiro, Bruce R. Munson, David P. DeWitt. John Wiley, 2003*
- *Fundamentals of Engineering Thermodynamics. M. J. Moran, H.N, Shapiro, D. N, Boettner, M. B. Bailet. John Wiley, 2018*
- *Fluid Mechanics. F.rank M. White. McGraw Hill, 2009*
- *Fundamentals of Heat and Mass Transfer. F. P. Incropera, D. P. Dewitt, T. L. Bergman, A. S. Lavin. John Wiley, 2007*









Why Thermodynamics?

Science of energy transfer and conversion

Life is nothing but: exchanges or transformations!

Material ↔ **Im – Material**

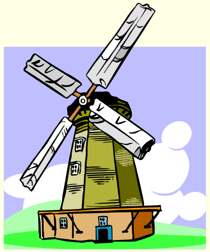
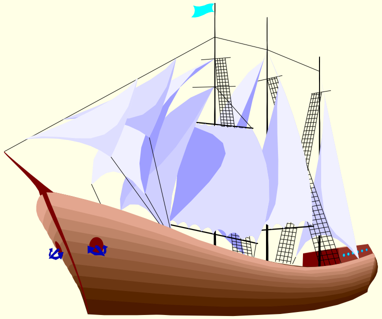
Knowledge
Feelings …

Mass
Charge
Force
…
Energy

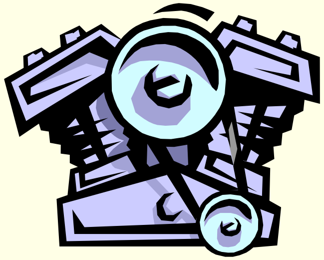
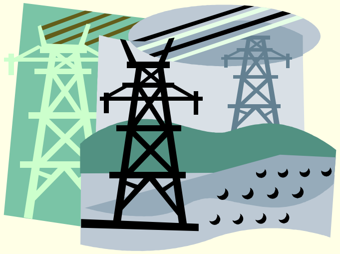
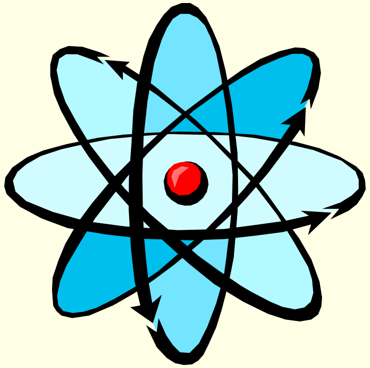
Energy is not only calorific or mechanical!

History of Energy

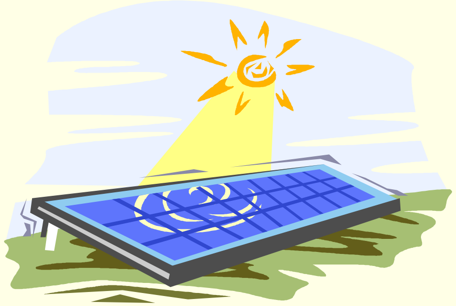
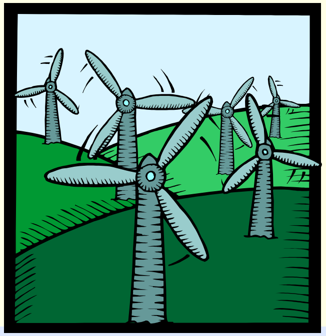
Pre-Industrial

Industrial

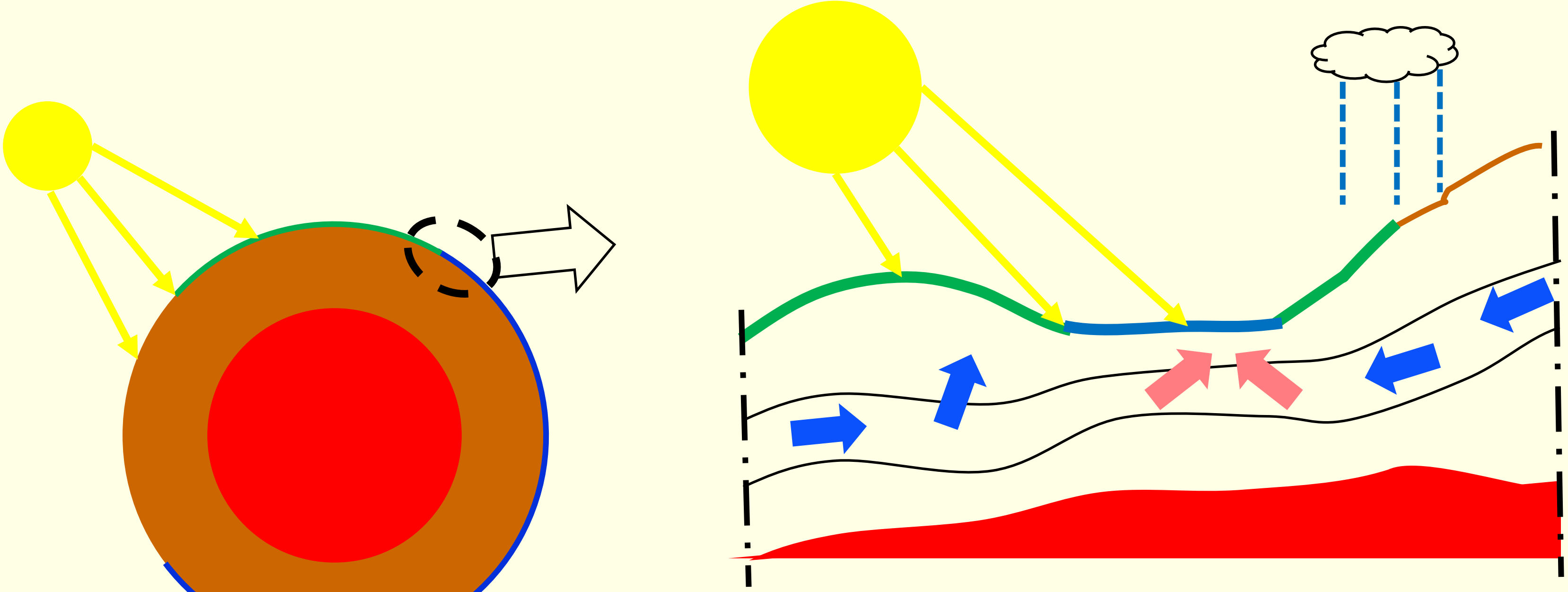





New Energies

Geothermal, Biomass, Waves, …

Geothermal energy: Source, Applications



Can be used to: Produce electricity, to heat, or to cool

Need to understand:

- ***How energy changes its form? (thermodynamics)***
- ***How water moves? (fluid mechanics)***
- ***How heat moves? (heat transfer)***

Place of thermodynamics academically

Basic Sciences

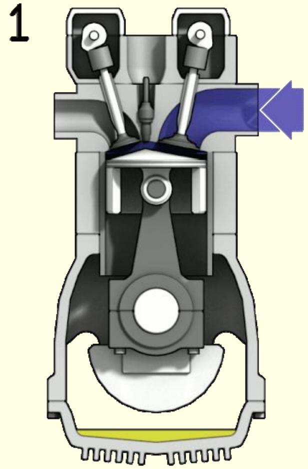
Math – Physics
Mechanics – Chemistry

Basic Engineering Sciences

Thermodynamics
Heat and mass transfer – Fluid mechanics
…

Physics of semi-conductors

Applied Engineering Sciences



Internal
Combustion
Engines

Power plants

Refrigeration
& Air Conditioning

Chemical industries …

To retain of chapter 1

- ***Course objectives and contents***
- ***Life depends on energy***
- ***Human development is related to energy***
- ***Energy crisis – environmental impact:***
  - ⇒ ***Renewable energies***
  - ⇒ ***Geothermal***
- ***Applications of thermodynamics are endless***

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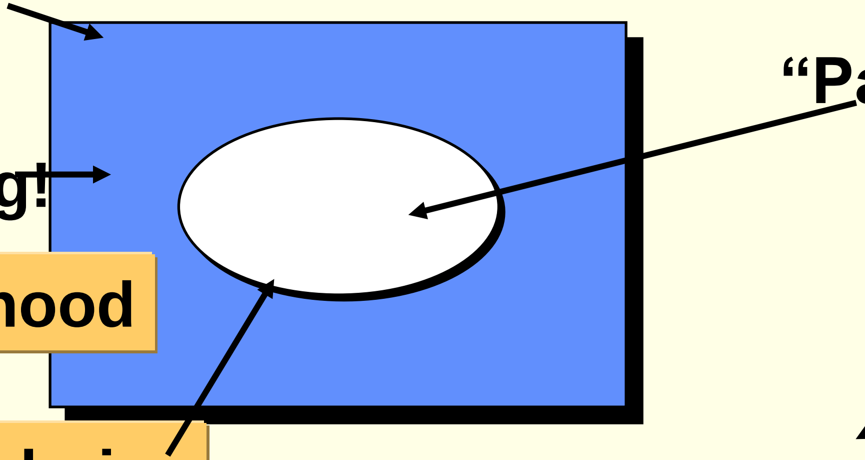
Terminology: “System”



“All” Universe!

Remaining!

Neighborhood

Boundaries

“Part” of interest

A system

Part = materiel portion?

control mass

Part = space zone?

control volume

	isolated system	closed system	open system
Energy	no	crosses	crosses
Matter	no	no	crosses

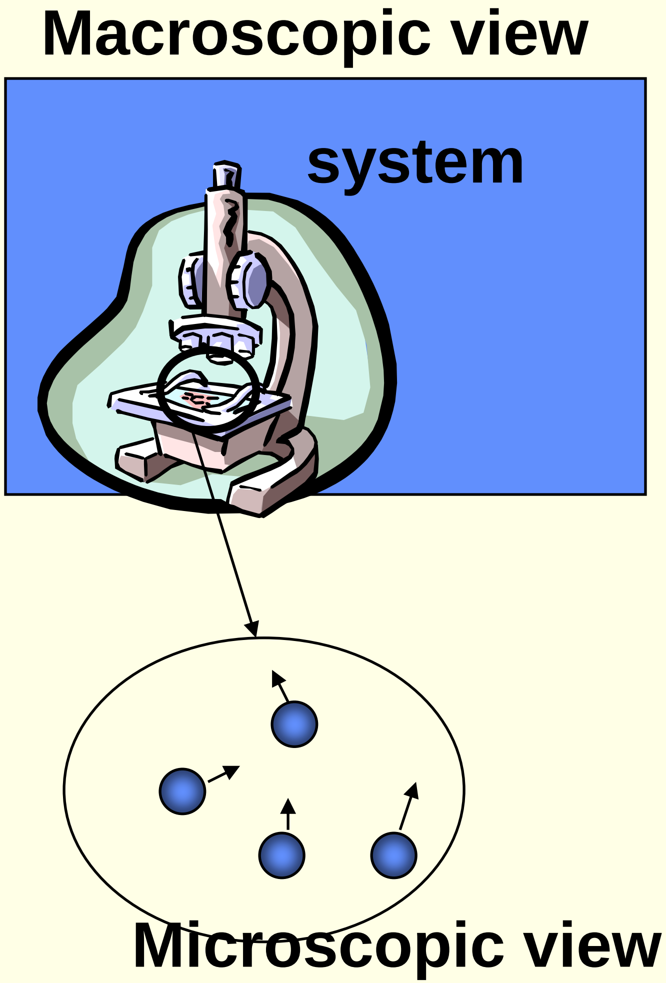
Macro or Micro?

- **Classical Thermodynamics**

  Matter is a continuum
  Essential part of the course

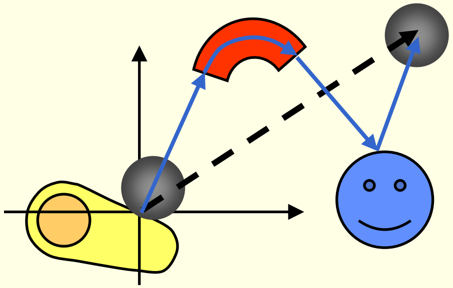
- **Statistical Thermodynamics**

  Matter is discrete
  To better understand



State

The state **of a system:**
Its nature and characteristics
AT a given instant



Distance along path (trajectory)
Distance from origin (state)

State Properties :
Quantitative description of the state
AS IS,
NOT how did the system reach it (the trajectory)

State property ≠ Property of the trajectory

Additional

Equilibrium

***Equilibrium*:**
Inability to perform a spontaneous change of state

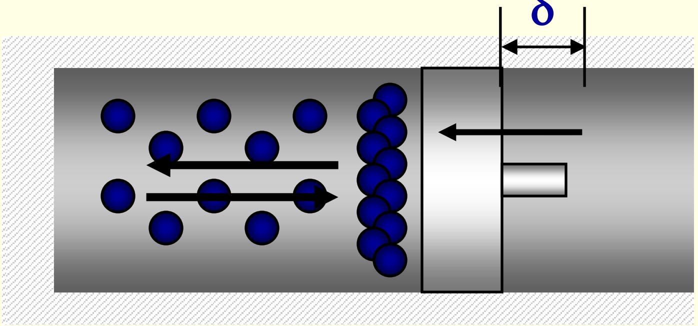
State properties of a system CANNOT be defined outside EQUILIBRIUM

Pressure of a system ⟺ Uniform Pressure

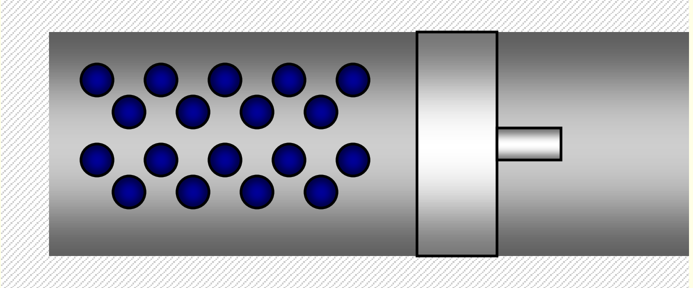
Process

Process **: continuous transition from state A to state B**

A process in “equilibrium”?!!



A process in “quasi- equilibrium”



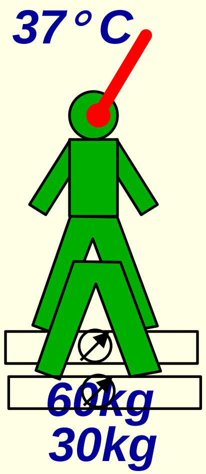
Cycle :

A set of processes ending by the initial state

State Properties

➢ ***Extensive Property :***

- **Proportional to size: Mass, volume, energy, charge, …**



➢ ***Intensive Property :***

- **Independent of size (same value at each portion):**
  Temperature, pressure, electric potential, …

➢ ***Specific Property :***

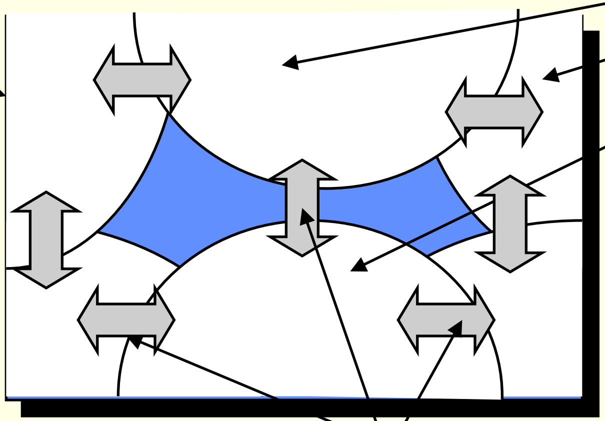
- **Ratio of two Extensive Properties:**
  specific volume (=1/Density), specific energy, …

Internal vs external Property : (Temperature vs speed)

Additional

Concepts and Laws

The world is unified (and unique!)
But, to understand it, … humans have to «analyze it»



Define concepts

Define relations between concepts

"All" the universe again!

relations

Ex:
Charges – Attraction Force
Mass – Force – Laws of Newton

A Principle (Law)	- **A fact proven by experience** - **Other theorems are deduced from principles**

Dimension versus unit

Dimension	**Units**
Distance	**meter, inch, foot, …**
Time	**second, hour, day, month, …**
Mass	**kilogram, pound, ton, …**

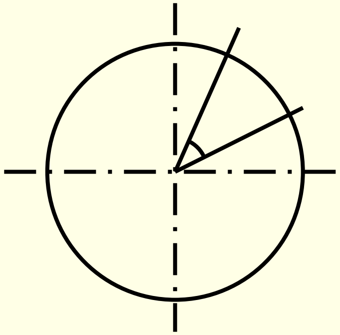
SI system

Basic SI Units:

Length	meter	m
Mass	kilogram	kg
Time	second	s
Electr. Current	Ampere	A
Temperature	Kelvin	K
Q. of matter	mole	mol
Lumin. Intensity	candela	cd

Sup. SI Units :

Plane Angle	Solid Angle
radian	steradian
rad	sr



Derived SI Units :

Force	Newton	N	$= \text{kg m/s}^2$
Pressure	Pascal	Pa	$= \text{N / m}^2$
Energy	Joule	J	$= \text{N m}$
Power	Watt	W	$= \text{J / s}$

…

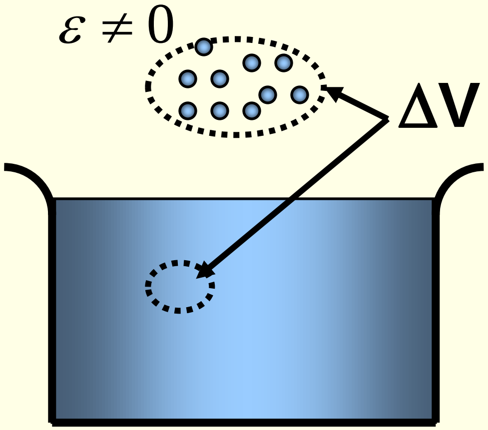
Multiples of Units:

M	Mega 10^6
G	Giga 10^9

…

Density & specific volume

Definition of Density: $\boxed{\rho \equiv \lim_{\Delta V \to \varepsilon > 0} \frac{\Delta m}{\Delta V}}$



Specific (Intensive) Property

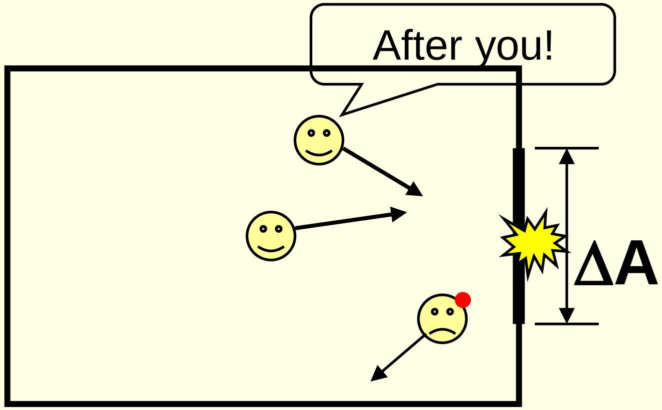
Molecular Mass	μ = mass of a mole (in g/mol or kg/kmol)	
Molar Density	$\bar{\rho} = \rho / \mu$	SI Units : kmol/m^3
Specific Volume	$v = 1/\rho$	SI Units : m^3/kg
Molar Specific Volume	$\bar{v} = v\,\mu$	SI Units : m^3/kmol

Defining Pressure

Force due to the impact of molecules on the wall

Definition of pressure: $$\boxed{P = \lim_{\Delta A \to \varepsilon > 0} \frac{\Delta F}{\Delta A}}$$

$$\varepsilon \neq 0$$



Specific (Intensive) Property

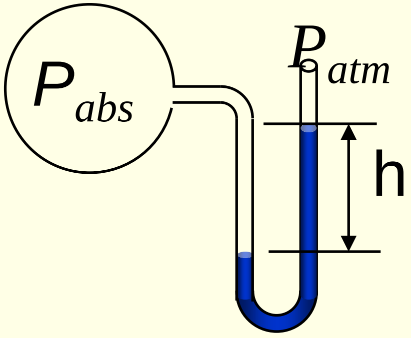
SI Units: Pa = N / m^2 = kg/ms^2

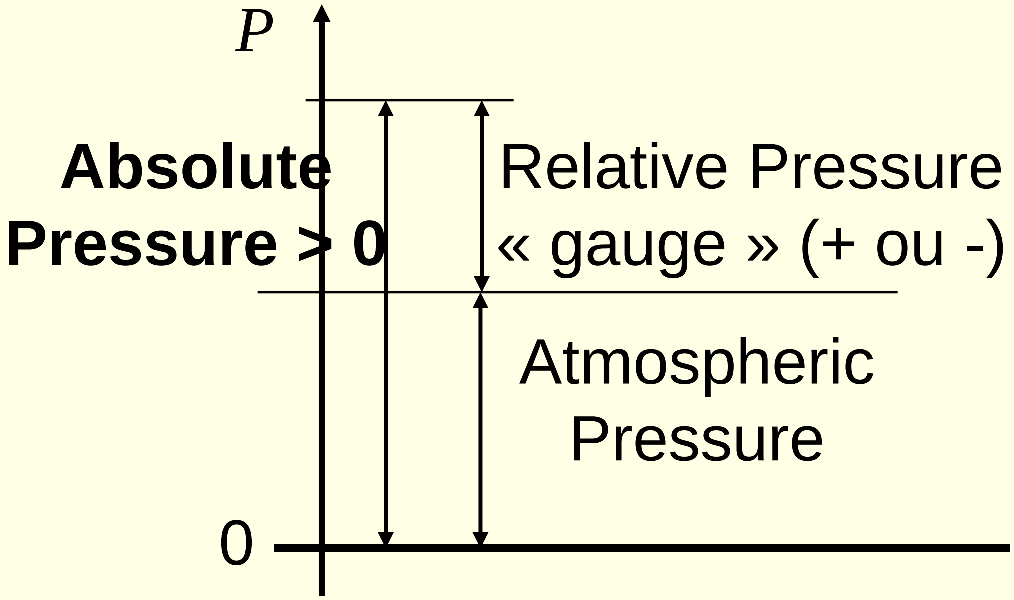
Other units
M Pa = 10^6 Pa;
bar =10^5 Pa;
atm = 1.013 bar;
psi = 1lb/in^2= 6.8948 kPa
torr = 1mmHg=133.32 Pa

Measuring Pressure

An example for low P

Manometer





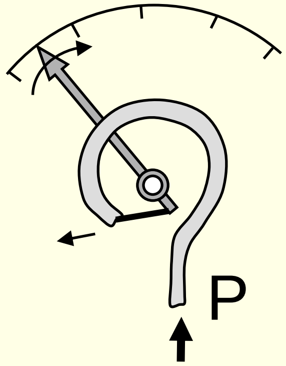
Absolute Pressure > 0

Relative Pressure « gauge » (+ ou -)

Atmospheric Pressure

$$P_{abs} - P_{atm} = P_{gauge} = \Delta\rho\, g\, h$$

An example for high P: **Bourdon**



to measure P_{atm}:

Barometer:

P_{atm} **against vacuum**

What is Temperature?!

Intensive Property

Temperature is a measure of the ability:

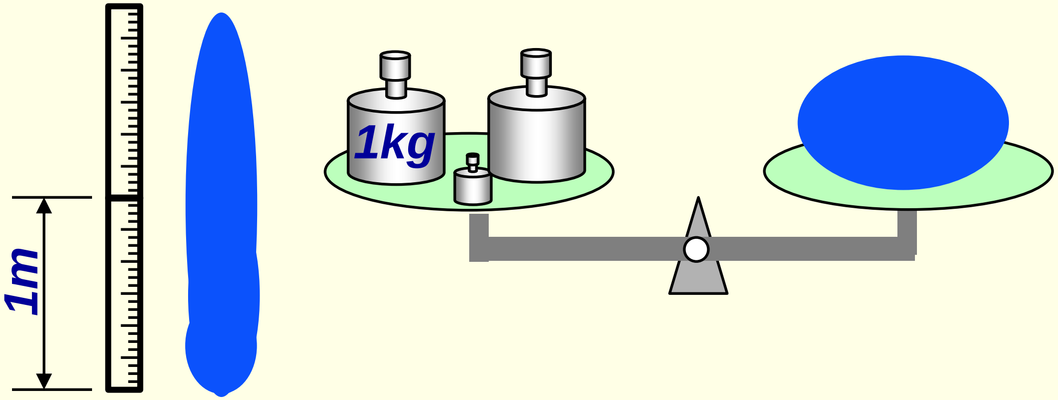
chaotic

macro: ***to transfer* calorific *energy***

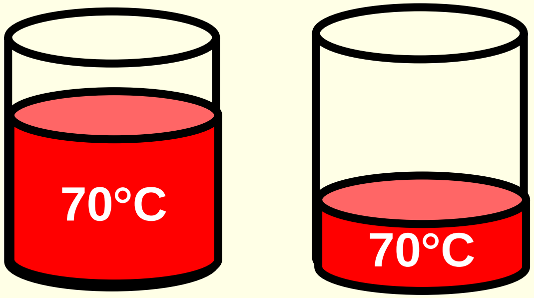
micro: **of molecules to transmit their *excitation* *intensity***

Problematic of an objective scale for an *intensive* property

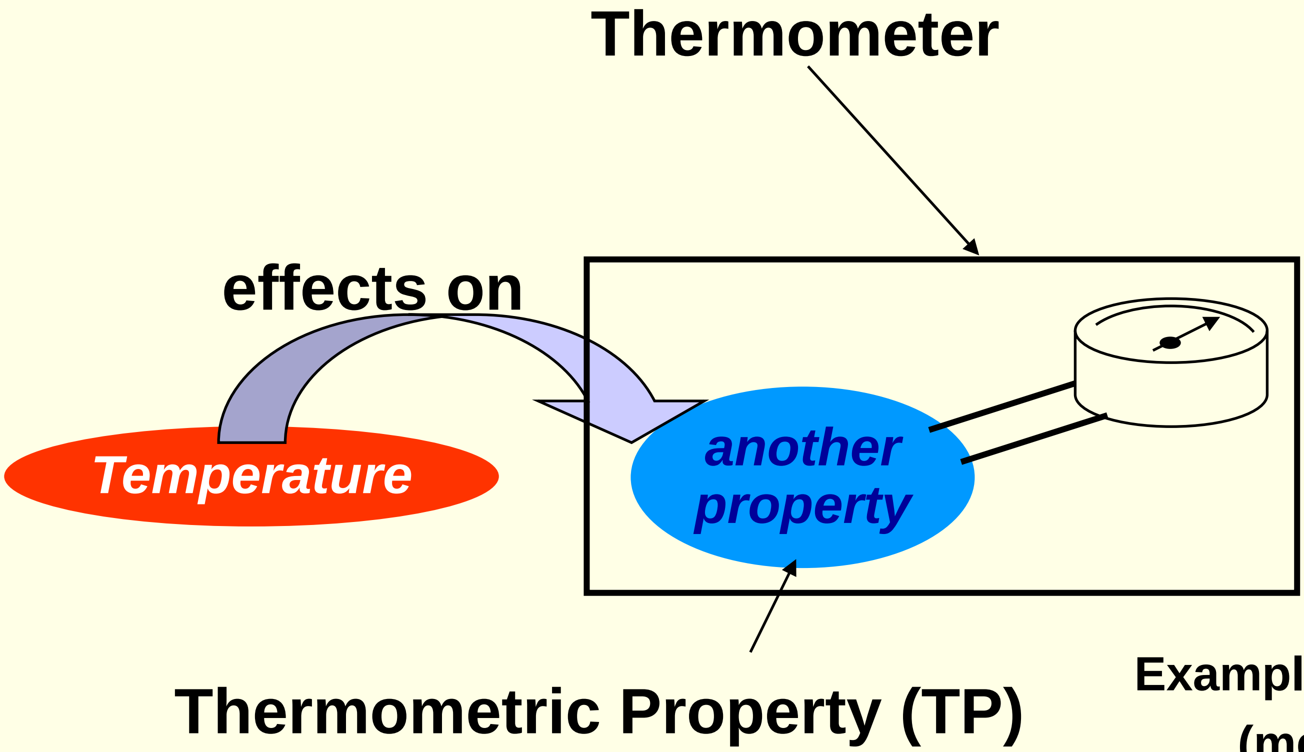
Extensive property is easy to measure:



Intensive property is difficult:



What is a Thermometer?



Example : the volume of a liquid (mercury thermometer)

Question :
What is the relation:
Thermometric Property (TP) $\Rightarrow$ T?!

The "Zero'th" Law!

Defining equality :

Thermal

If **Body A** ⟺ **Body B**

Equilibrium

And **Body A** ⟺ **Body C**

Then **Body B** ⟺ **Body C**

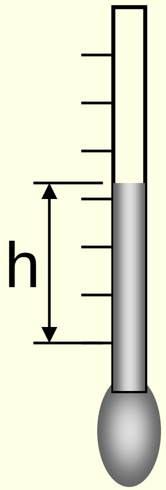
Defining difference :

Hence, we can define an *arbitrary* temperature scale :

Colder → Hotter

For a *rational* scale :
The thermodynamic temperature scale (see below)

Thermometers

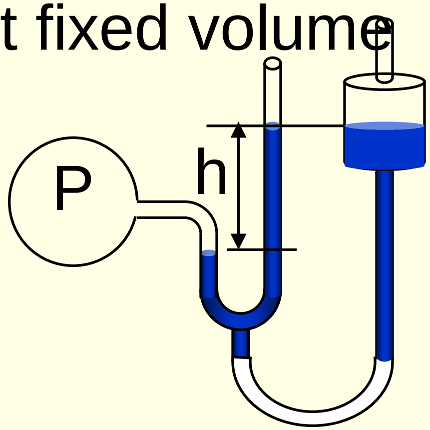


$T=ah+b$

Mercury Thermometer

fixed points

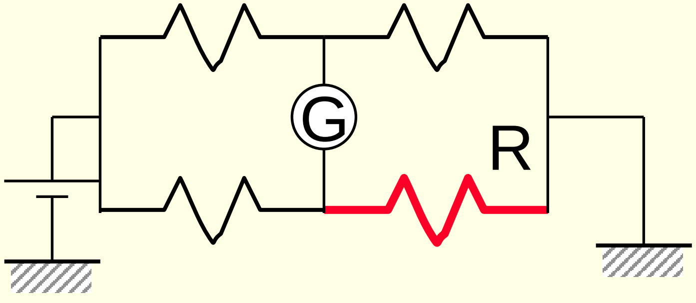
Pressure of a gas at fixed volume



$T = aP$

Ideal Gas Thermometer ***

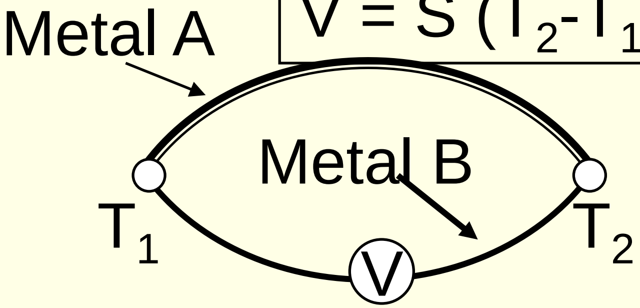
Resistance of a metal (Platinum)



$R = a + bT + cT^2$

Resistance Thermometer

$\boxed{V = S\,(T_2 - T_1)}$



Thermocouple

Importance of assumption!

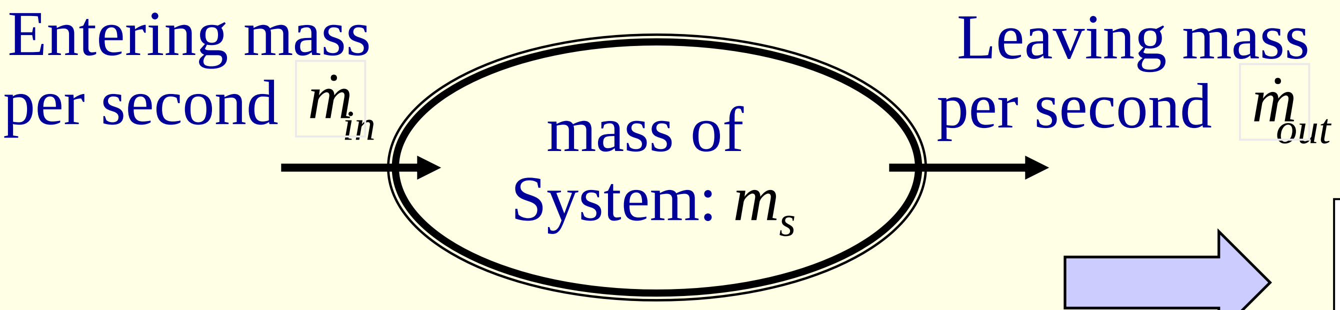
$$t^* = 100\,\frac{\left(v - v|_0\right)}{\left(v|_{100} - v|_0\right)}$$

v specific volume (cm^3/g)

t^* Temperature **assuming linear** in v (°C)

Gas Thermo-meter	Water Therm.		Alcohol Therm.		Mercury Therm.	
t	v	t^*	v	t^*	v	t^*
-25	-	-	1.2167	-18.8	0.073220	-25.0
0	1.0002	0	1.2475	0	0.073556	0
25	1.0029	6.2	1.2800	19.8	0.073890	24.9
50	1.0121	27.5	1.3170	42.4	0.074225	49.9
75	1.0259	59.4	1.3604	68.8	0.074561	74.9
100	1.0435	100	1.4116	100	0.074898	100

Conservation of mass



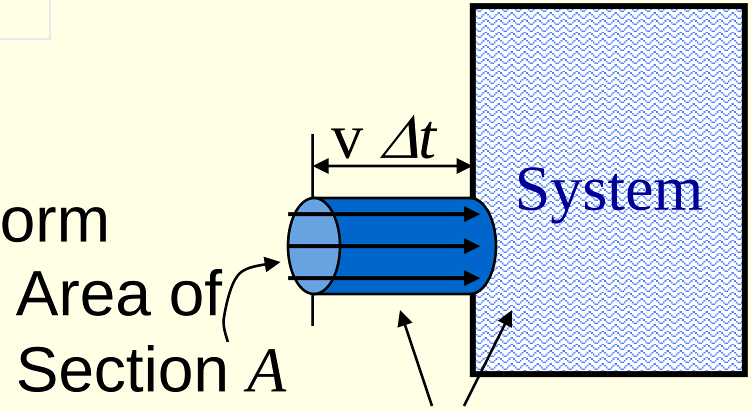
- **Not relativistic!**

$$\boxed{\frac{d\,m_s}{dt} = \dot{m}_{in} - \dot{m}_{out}}$$

If the density ρ was uniform then $m_s = \rho V$
(V is the volume) otherwise: $m_s = \int \rho\, dV$

To find $\dot{m}$

If ρ and the normal speed *v* were uniform
then $\dot{m} = \rho A (\mathrm{v}\, \Delta t) / \Delta t = \rho\, \mathrm{v}\, A$
otherwise: $\dot{m} = \int \rho \mathrm{v}\, dA$



Volume entering
during the time Δt = v $\Delta t\, A$

To retain of chapter 2

- **System:** Control mass (closed), Control volume (open), Isolated
- **State property:** *(NOT trajectory), intensive vs extensive, specific*
- **World unity:** Relation between concepts and laws, principles
- **Dimensions and units:** SI 7 basic units, derived units
- **Specific volume:** (specific) $v = 1 / \rho$ (per unit mass or unit mole)
- **Pressure:** (specific) *absolute (>0), gauge, units*
- **Temperature:** *(intensive – difficulty), 0^{th} Law, thermometric properties, fixed points, thermometers, ideal gas thermometer*
- **Conservation of mass:**

$$\boxed{\frac{d\,m_s}{dt} = \dot{m}_{in} - \dot{m}_{out}}$$

$$\dot{m} = \int \rho\, v\, dA$$

$= \rho\, v\, A$ *if uniform*

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Energy: definition, simple forms

To define energy:

Define specific forms

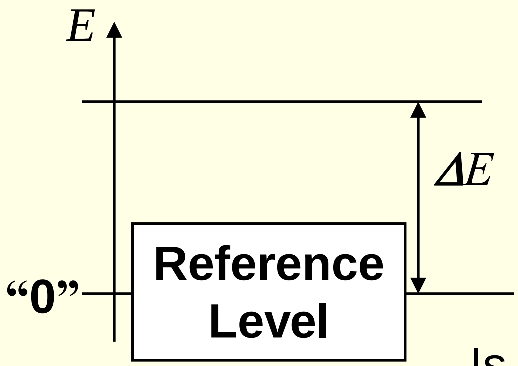
Generalize

Conservation Law

The concept of ENERGY

~ Capacity to perform a change

It is only Δ that counts!



Energy is classified into

Energy crossing boundaries

Energy stored in the system

Is characterized by the **Power** (Rate of energy transfer) i.e. in **Watt** = Joule/s

Is measured in **Joule**

Different forms

Forms of energy:

- Mechanical (work)
- Internal
- Calorific (heat)
- Electric, …

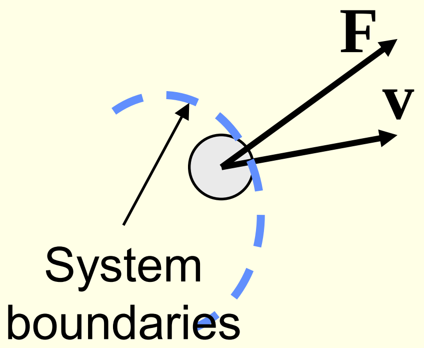
Will be treated
In some
details

- Chemical,
- Nuclear,
- Optic,
- Acoustic,
- Magnetic, ...

Other forms

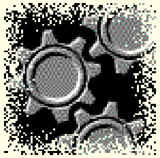
Mechanical Power and Work

Mechanical power is: Power **crossing** the boundaries of a system due to the action of a **force** that **causes a displacement velocity**



$\mathbf{F}$ Force exerted **on** the system New convention

~~Force exerted **by** the system Classic convention~~

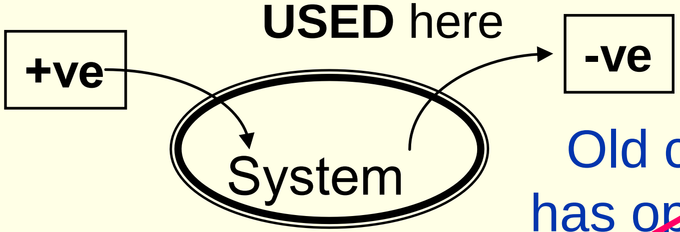


$\mathbf{v}$ Displacement velocity of **matter** due the force

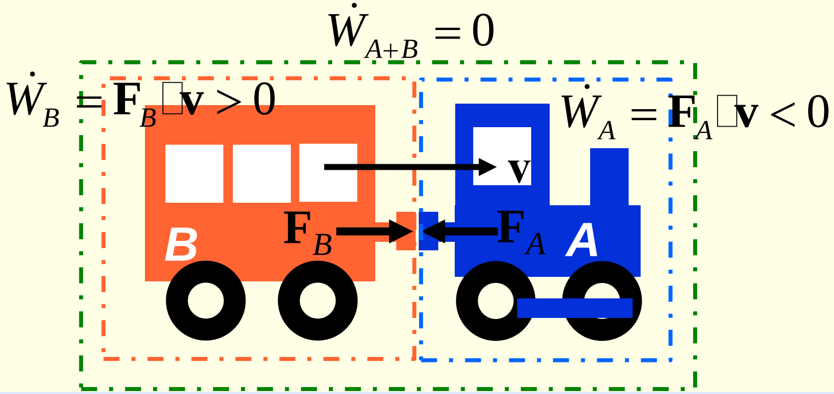
Mechanical Power : $\dot{W} = \mathbf{F} \bullet \mathbf{v}$

Mechanical Work : $W = \int_{t_1}^{t_2} \dot{W}\, dt = \int_{t_1}^{t_2} \mathbf{F} \bullet \mathbf{v}\, dt = \int_{\vec{\mathbf{x}}_1}^{\vec{\mathbf{x}}_2} \mathbf{F} \bullet d\mathbf{x}$

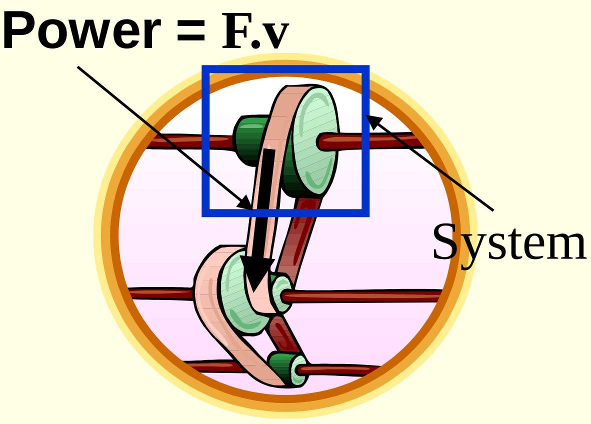
New Sign Convention for work **USED** here

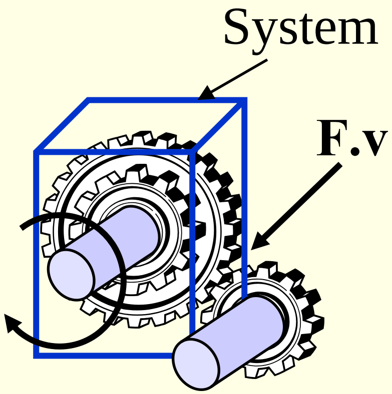


~~Old convention has opposite signs~~



Examples of point forces





Power = F v = F $(r\,\omega)$ = $T\omega$

Torque * angular speed

$\mathbf{F}_{ext}$ Force **on** the system

$\mathbf{F}_{sys} = -\mathbf{F}_{ext}$ Force **by** the system

Work of a force causing acceleration

$$\mathbf{F}_{ext} = -\mathbf{F}_{sys} = m\,d\mathbf{v}/dt$$

$$W_s = \int_{t_1}^{t_2} \left(m\,d\mathbf{v}/dt\right) \bullet \mathbf{v}\,dt$$

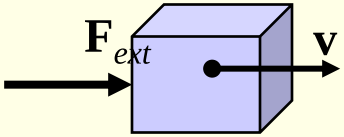
$$W_s = \int_{v_1}^{v_2} m\mathbf{v} \bullet d\mathbf{v}$$

$$W_s = \frac{1}{2} m\left(\mathrm{v}_2^2 - \mathrm{v}_1^2\right)$$

Kinetic Energy :

linear ½ $m\,\mathrm{v}^2$

rotational ½ $I\omega^2$



Work of a force against Gravity

$$\mathbf{F}_{ext} = -\mathbf{F}_{sys} = -m\,\mathbf{g}$$

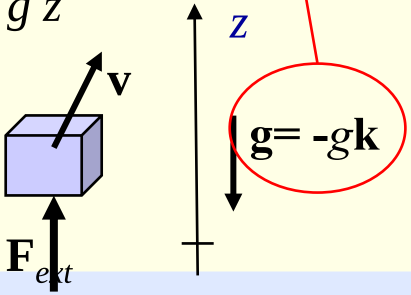
$$W_s = \int_{t_1}^{t_2} \left(-m\,\mathbf{g}\right) \bullet \mathbf{v}\,dt$$

$$W_s = \int_{z_1}^{z_2} mg\,dz$$

$$W_s = m\,g\left(z_2 - z_1\right)$$

Potential Energy :

$m\,g\,z$



Work by an elastic body

F is the force **on** the system= $k\,\mathbf{x}$

x is the ***elongation***
(= Length – Free length)

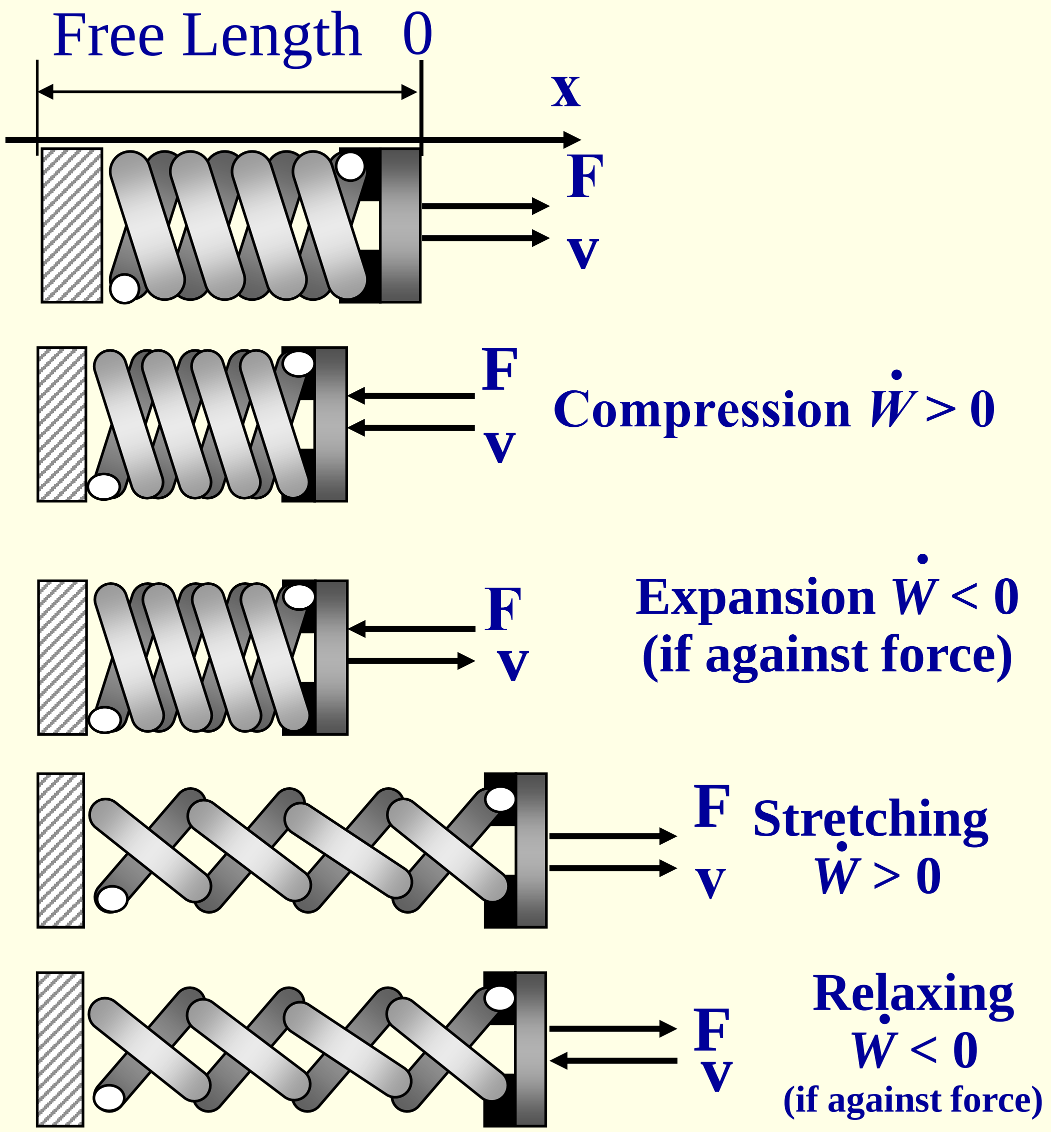
v is the ***materiel*** velocity

Power $\dot{W} = \mathbf{F} \cdot \mathbf{v}$

Work $W = \int_{t_1}^{t_2} \underbrace{\mathbf{F}}_{} \bullet \underbrace{\mathbf{v}\,dt}_{}$

$$W = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \overbrace{k\mathbf{x}} \bullet \overbrace{d\mathbf{x}}$$

$$W = \frac{1}{2} k \left(x_2^2 - x_1^2 \right)$$



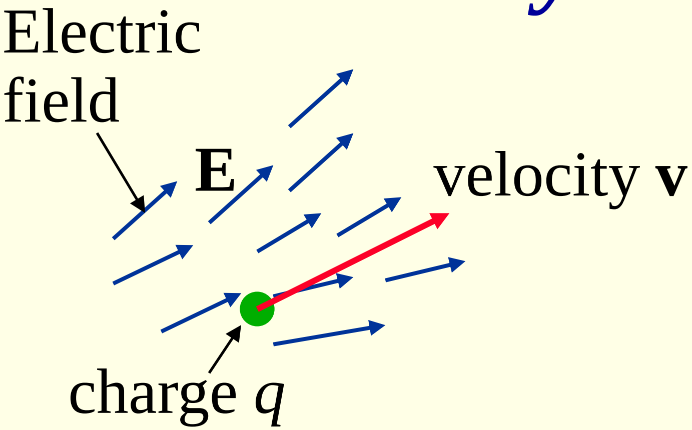
Compression $\dot{W} > 0$

Expansion $\dot{W} < 0$
(if against force)

Stretching
$\dot{W} > 0$

Relaxing
$\dot{W} < 0$
(if against force)

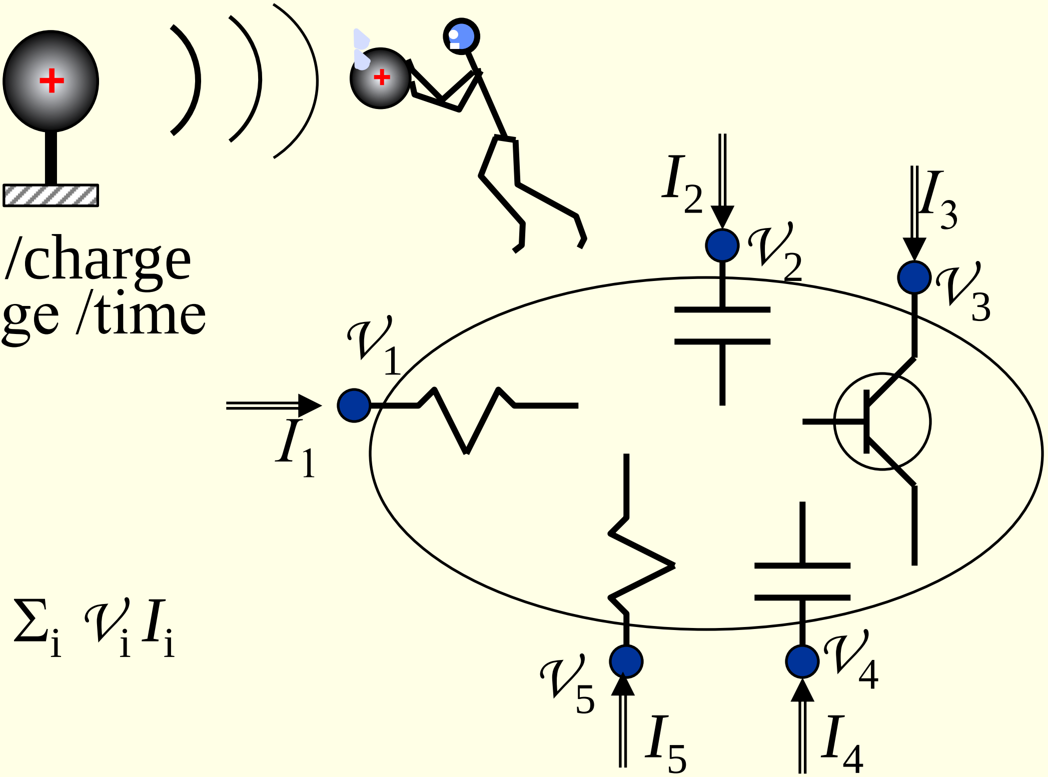
Work by an electric field on a charge



Power = $-\,q\,\mathbf{E}\,.\,\mathbf{v}$

Work $W = -\int_{t_1}^{t_2} q\mathbf{E}\bullet\mathbf{v}\,dt$ $\quad W = -\int_{\mathbf{x}_1}^{\mathbf{x}_2} q\mathbf{E}\bullet d\mathbf{x}$

Electric Potential $\mathcal{V}(\mathbf{x}) = W/q = -\int_{\infty}^{\mathbf{x}} \mathbf{E}\bullet d\mathbf{x}$ **(Volt)**



Potential at node 1: $\mathcal{V}_1$ = work /charge

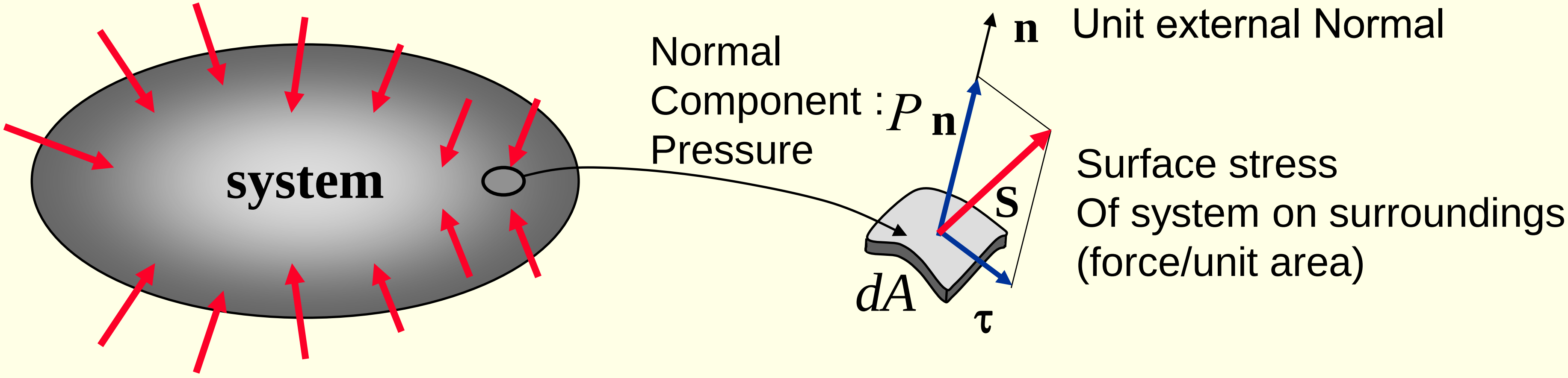
Current entering node 1: I_1 = charge /time

Power entering node 1: $\mathcal{V}_1 I_1$

Power received by the circuit: $\Sigma_i\ \mathcal{V}_i I_i$

Work $W = \int \mathcal{V} I\,dt$

Work of distributed forces
A – Over a surface



system

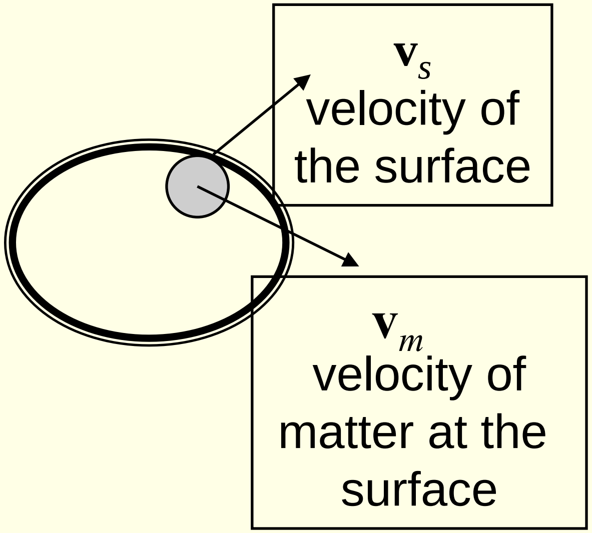
Normal Component : Pressure $P\,\mathbf{n}$

$\mathbf{n}$ Unit external Normal

$\mathbf{S}$ Surface stress Of system on surroundings (force/unit area)

dA

$\boldsymbol{\tau}$



$\mathbf{v}_s$ velocity of the surface

$\mathbf{v}_m$ velocity of matter at the surface

Closed System : $\mathbf{n.v_s = n.v_m}$

Open System : $\mathbf{n.v_s \neq n.v_m}$ (matter crosses)

Power : $\dot{W} = -\int_A \mathbf{S}\cdot\mathbf{v}_m\,dA$

Work : $W = \int_{t_1}^{t_2}\left(-\int_A \mathbf{S}\cdot\mathbf{v}_m\,dA\right)dt$

Case 1: Work of Changing Volume

If:

- No friction ($\mathbf{S} = P\,\mathbf{n}$)
- Equilibrium (pressure P uniform)
- Closed system ($\mathbf{n}.\mathbf{v}_s = \mathbf{n}.\mathbf{v}_m$)

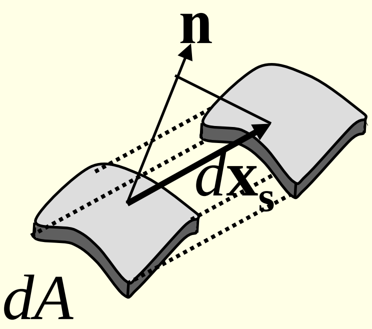
Then:

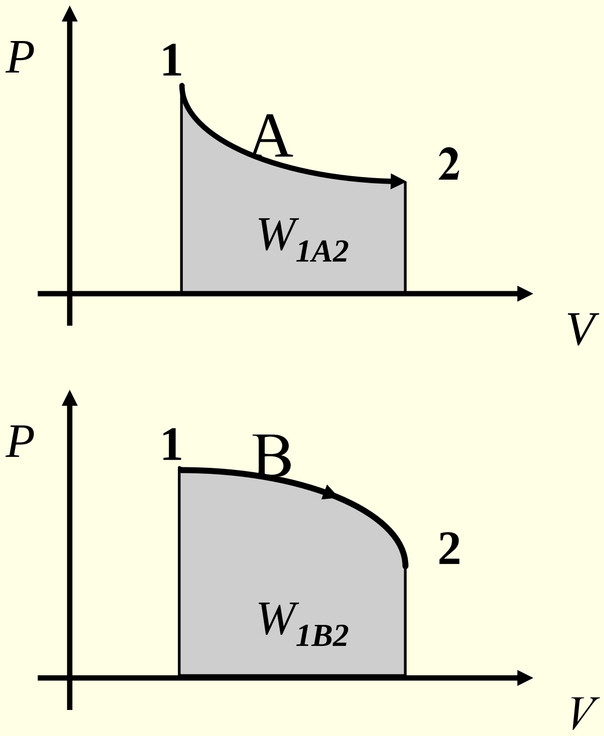
$$W = \int_{t_1}^{t_2} \left(-\int_A \mathbf{S}\cdot\mathbf{v}_m\, dA \right) dt$$

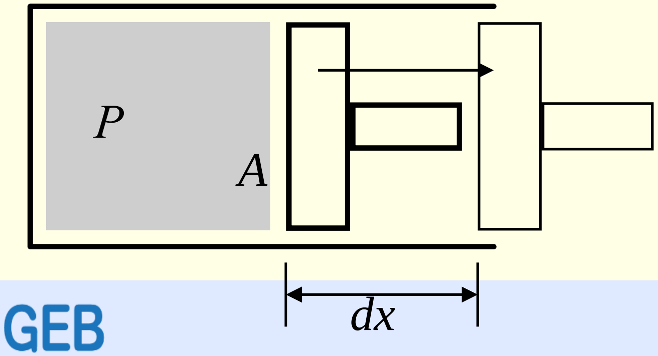
$$\Rightarrow W = \int_{t_1}^{t_2} \left(-P \int_A \left(\mathbf{n} \cdot \frac{d\mathbf{x}_s}{dt} \right) dA \right) dt$$

$$\Rightarrow W = \int_{t_1}^{t_2} -P \frac{dV}{dt}\, dt$$

$$\Rightarrow \boxed{W = -\int P\,dV}$$







$$\boxed{W = -\int F\,dx}$$

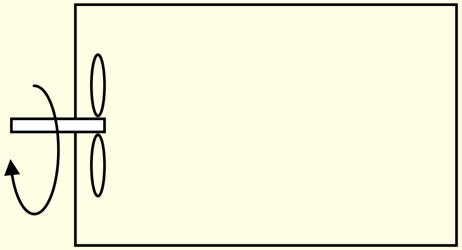
$$\boxed{W = -\int PA\,dx = -\int P\,dV}$$

NB: W **depends on states 1,2**
AND the PATH
$W_{1A2} \neq W_{1B2}$

Work of changing volume is NOT always $-\int PdV$

N.B. 1:- $\int P\,dV$ **is not always work !**

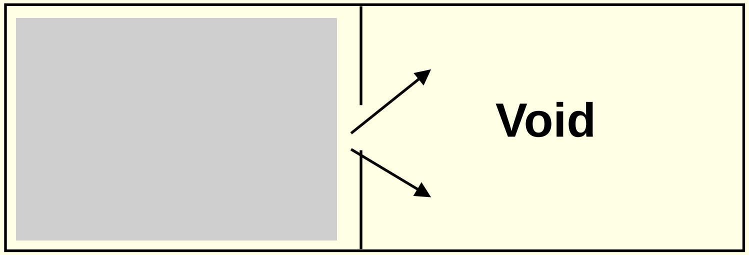
Fan work



$$-\int P\,dV = 0$$
$$W > 0$$

Since $dV = 0$

Free expansion



$$-\int P\,dV < 0$$
$$W = 0$$

The gas expands against void:

⇒ No exterior force ⇒ No work!

N.B.:

In the ideal case (no friction) work is called W_{ideal}.

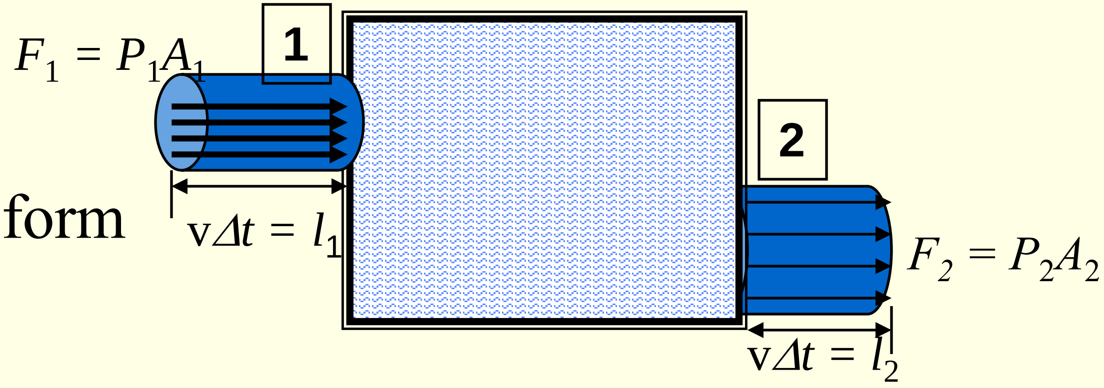
Evidently: $W_{actual} - W_{ideal} > 0$

Whether work is received by or given to the system

Case 2: Flow work

If:

- No friction ($\mathbf{S} = P\,\mathbf{n}$)
- Equilibrium (pressure P is uniform over each section)
- Open System ($\mathbf{n.v}_s \neq \mathbf{n.v}_m$)
- Steady state (independent of time)



To force the fluid at entry:

$W_1 = P_1\,A_1\,l_1 = P_1\,V_1$

At exit: $W_2 = -\,P_2\,V_2$

Then:

$$W = \int_{t_1}^{t_2}\left(-\int_A \mathbf{S}\cdot\mathbf{v}_m\,dA\right)dt$$

$$W_{flow\,in} = \left(\int_{t_1}^{t_2} dt\right)\int_A (-P\mathbf{n})\cdot\mathbf{v}_m\,dA$$

$$W_{flow\,in} = -P\Delta t\int_A (\mathbf{n}.\mathbf{v}_m)\,dA$$

$$= P\Delta t\cdot(V/\Delta t) = PV$$

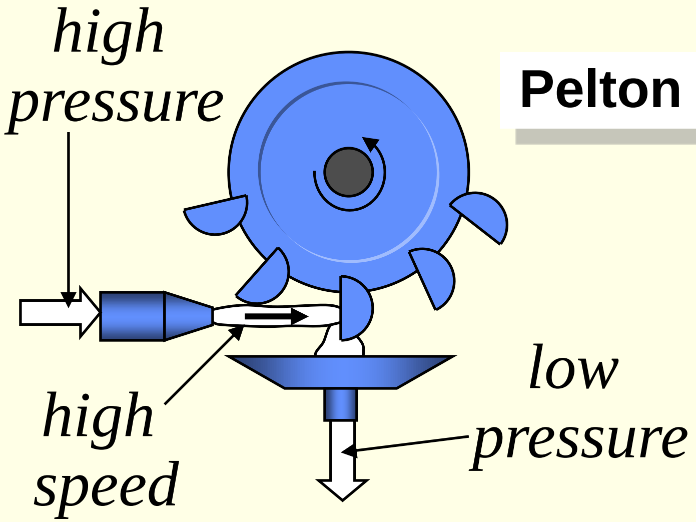
Flow work **=** $P_1\,V_1 - P_2\,V_2$

Useful work of an open system

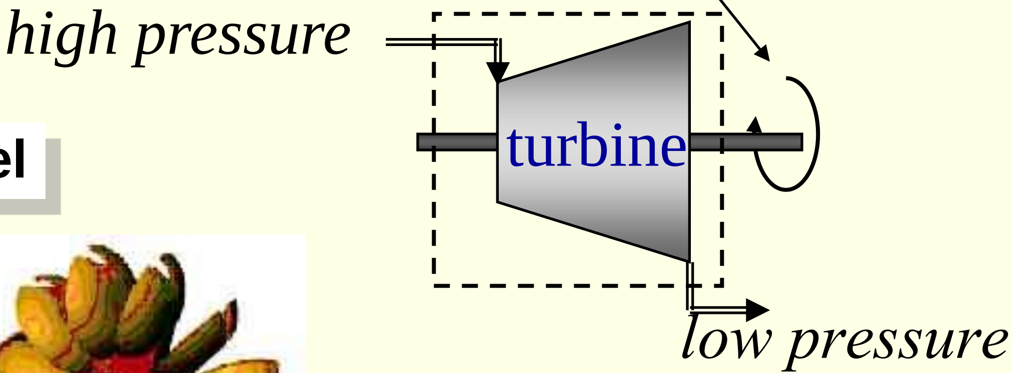
N.B.: $d\,PV = PdV + VdP$, **i.e.:** $\left(P_2V_2 - P_1V_1\right) = \int_1^2 PdV + \int_1^2 VdP$

$$\int_1^2 VdP = \underbrace{-\int_1^2 PdV}_{\text{Work of changing volume}} - \underbrace{\left(P_1V_1 - P_2V_2\right)}_{\text{Flow work}} =$$

Useful part of Work for a control volume



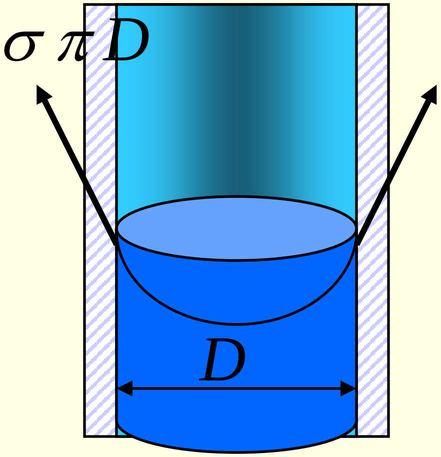
Pelton wheel



Work of Distributed Forces
B – Over a line

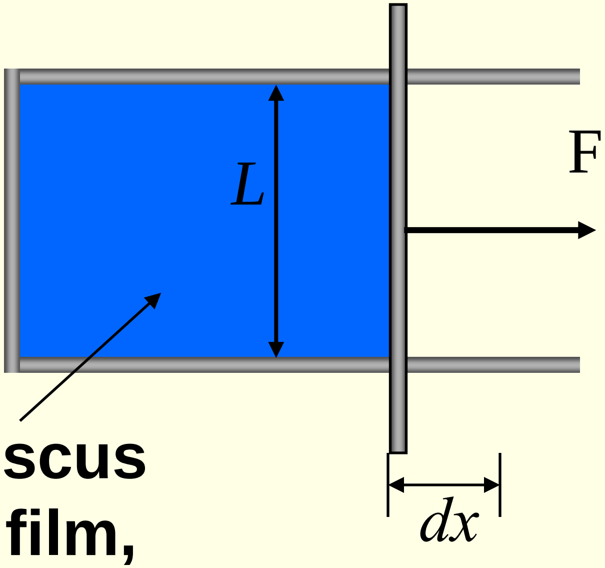
Additional

Coefficient of Surface Tension:
σ = Force / unit length



Force = $\sigma \pi D$

D



L

dx

Force = σL

Meniscus (thin film, e.g. soap)

Work on the *Line*

$$W = \int \sigma L \, dx$$
$$= \int \sigma \, dA$$

Internal Energy (qualitative)

Suppose we have N molecules each of mass m

Mass of body $M = m * N$

$\mathbf{v}_i$ is the absolute velocity of molecule i

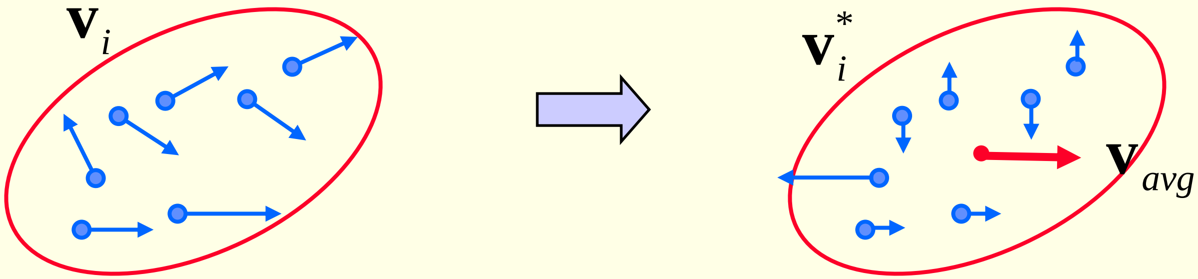
Body velocity
= average velocity

$$= \quad \mathbf{v}_{avg} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{v}_i$$

We can define: $\mathbf{v}_i = \mathbf{v}_{avg} + \mathbf{v}_i^*$

Deviation, vibration

By substituting

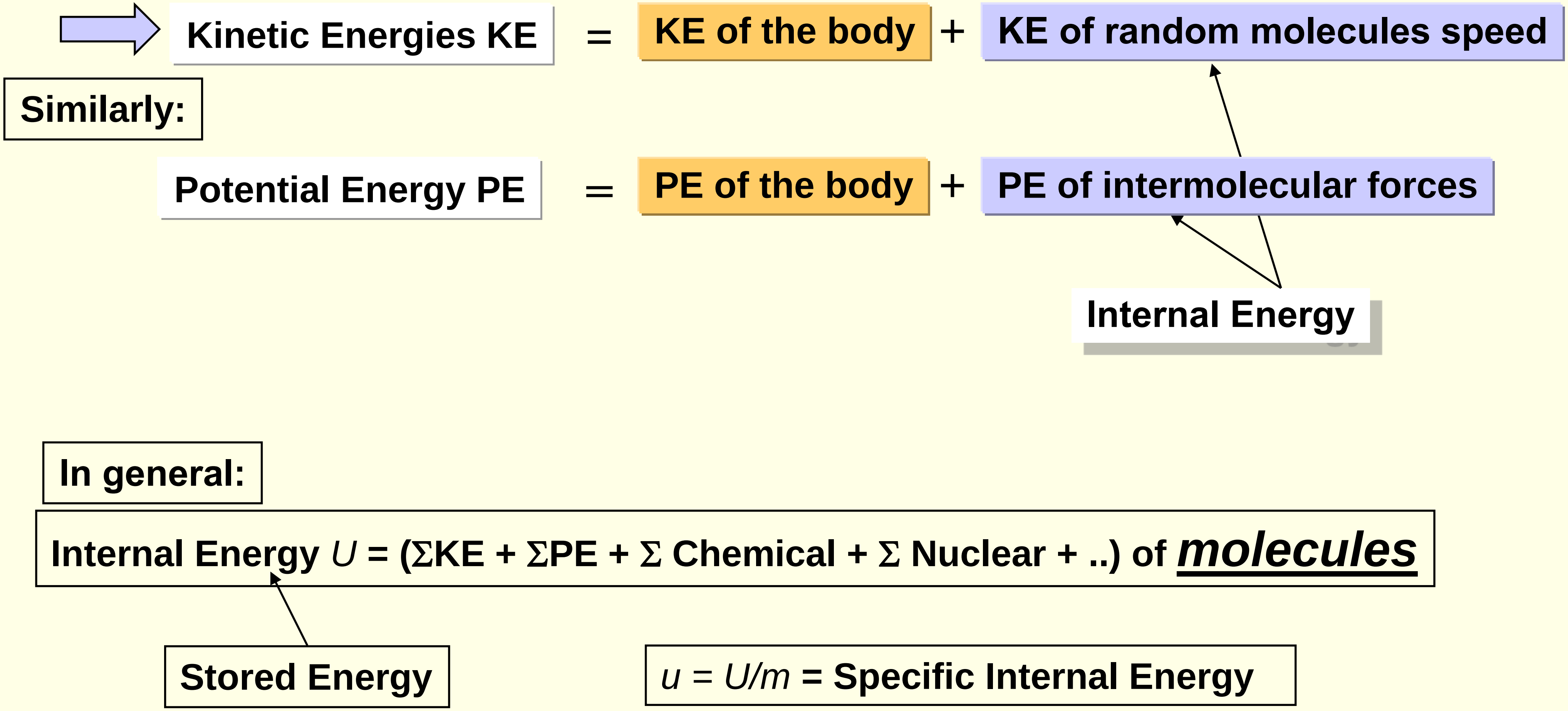


Kinetic Energy of N molecules:

$$KE = \frac{m}{2}\sum_{i=1}^{N}\mathbf{v}_i \cdot \mathbf{v}_i$$

$$KE = \underbrace{\frac{M}{2}\mathbf{v}_{avg}\cdot\mathbf{v}_{avg}}_{\text{KE of body}} + \underbrace{\frac{m}{2}\sum_{i=1}^{N}\mathbf{v}_i^*\cdot\mathbf{v}_i^*}_{\text{KE of molecules}} + \cancel{\underbrace{\mathbf{v}_{avg}\cdot m\sum_{i=1}^{N}\mathbf{v}_i^*}_{\begin{gathered}= 0\\ \text{(average of deviations)}\end{gathered}}}$$

Internal Energy : Generalization



Kinetic Energies KE = **KE of the body** + **KE of random molecules speed**

Similarly:

Potential Energy PE = **PE of the body** + **PE of intermolecular forces**

Internal Energy

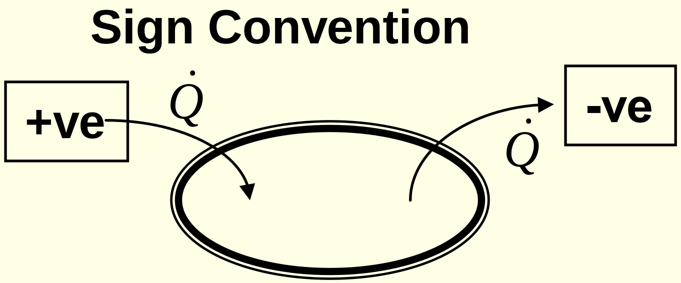
In general:

Internal Energy U = (ΣKE + ΣPE + Σ Chemical + Σ Nuclear + ..) of *molecules*

Stored Energy

$u = U/m$ **= Specific Internal Energy**

Heat



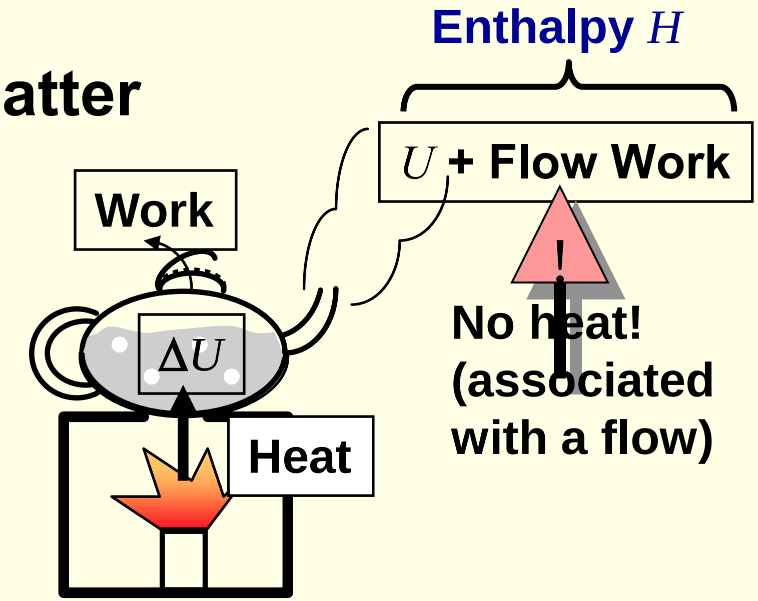
Heat power $\dot{Q}$ is:

- **A power that <u>crosses</u> the boundary**
- **Due to a temperature difference**
- **<u>NOT</u> associated with a transfer of matter**

Heat Power in Watt $\dot{Q}$

Heat : Q in Joule

System: Water in a tea pot.
What are energies involved?



No heat! (associated with a flow)

Distinguish between heat (energy that <u>crosses</u>) and internal energy (energy <u>stored</u> in a hot body)

Heat transferred is path dependent

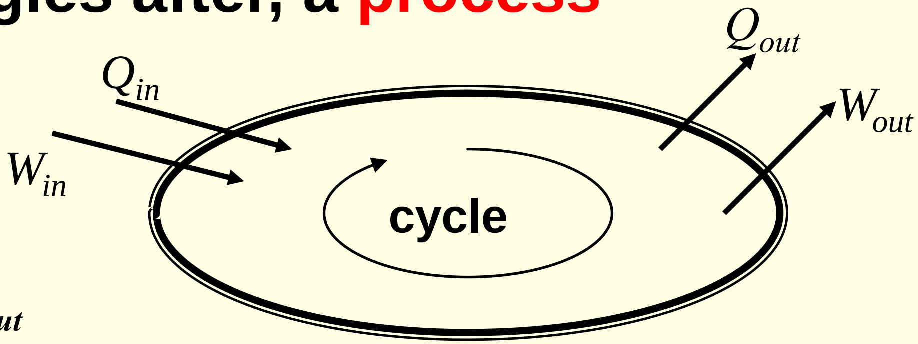
1^{st} Law for a Control mass

Energy is conserved

It can be transferred or transformed, but:

Σ Energies before = Σ Energies after, a process

For a control mass undergoing
a Complete cycle:



$$Q_{net} = Q_{in} - Q_{out} \qquad W_{net} = W_{in} - W_{out}$$

⇨ Final state = Initial state
Stored Energies before = Stored Energies after

⇨ $\boxed{\Sigma \text{ Crossing Energies} = Q_{net} + W_{net} = 0}$

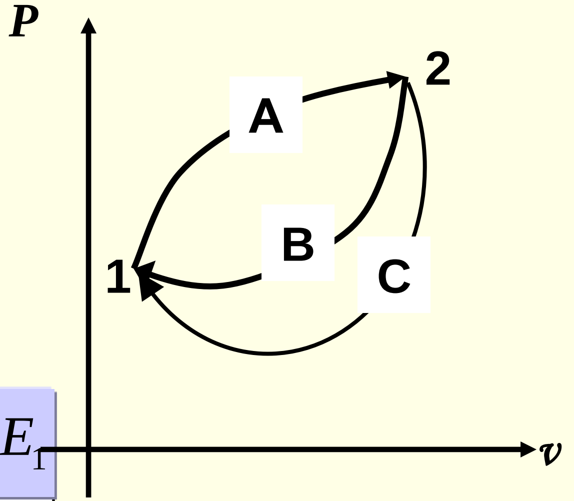
Total Internal energy E is a <u>State property</u>

Process:	**1A2**	**2B1**	**2C1**
Work:	$W_{1A2} = \int_{1A2} \dot{W} dt$	$W_{2B1} = \int_{2B1} \dot{W} dt$	$W_{2C1} = \int_{2C1} \dot{W} dt$
Heat:	$Q_{1A2} = \int_{1A2} \dot{Q} dt$	$Q_{2B1} = \int_{2B1} \dot{Q} dt$	$Q_{2C1} = \int_{2C1} \dot{Q} dt$

For the cycle 1A2B1: $\int_{1A2} \left(\dot{Q} + \dot{W} \right) dt + \int_{2B1} \left(\dot{Q} + \dot{W} \right) dt = 0$

For the cycle 1A2C1: $\int_{1A2} \left(\dot{Q} + \dot{W} \right) dt + \int_{2C1} \left(\dot{Q} + \dot{W} \right) dt = 0$

By subtracting: $\int_{2B1} \left(\dot{Q} + \dot{W} \right) dt = \int_{2C1} \left(\dot{Q} + \dot{W} \right) dt$



⇨ **For a *<u>process</u>*:**
The sum of energies crossing is *<u>independent</u>* of path:

$$\int_{12} \left(\dot{Q} + \dot{W} \right) dt = E_2 - E_1$$

$$Q_{12} + W_{12} = E_2 - E_1$$

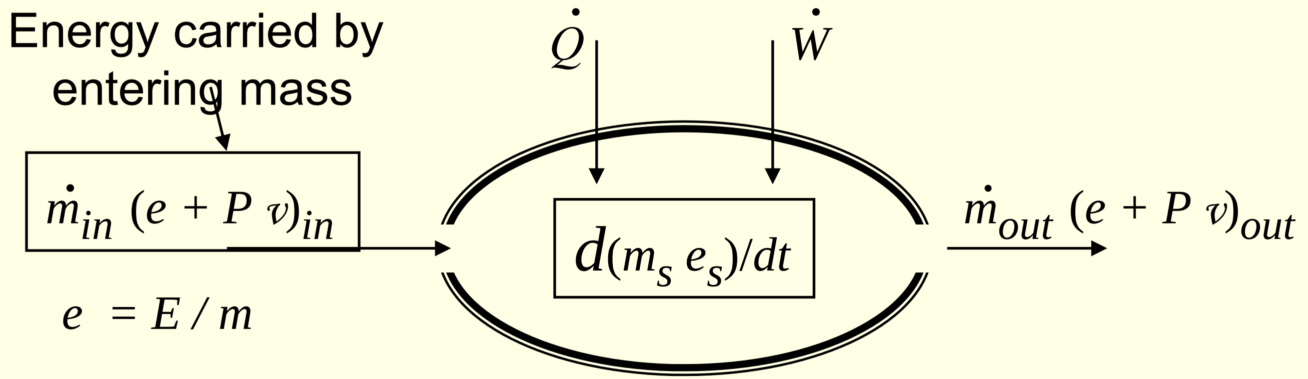
Total internal Energy

$$\Delta E = \Delta U + \Delta KE + \Delta PE + \ldots$$

$$\dot{Q} + \dot{W} = dE/dt$$

1^{st} Law : General form

Sum of received powers =rate of increase of stored energies



Energy carried by entering mass

$\dot{Q}$

$\dot{W}$

$\dot{m}_{in}\ (e + P\,v)_{in}$

$d(m_s\, e_s)/dt$

$\dot{m}_{out}\ (e + P\,v)_{out}$

$e = E/m$

$e + P\,v =$

$\boxed{(u + P\,v)} + KE + PE + ..$

$\boxed{h}$ Specific Enthalpy

$$\dot{Q} + \dot{W} + \dot{m}_{in}\left(h + KE + PE\right)_{in} - \dot{m}_{out}\left(h + KE + PE\right)_{out} = \frac{d}{dt}\left[m_s\left(u + KE + PE\right)_s\right]$$

$$Q + W + m_{in}\left(h + KE + PE\right)_{in} - m_{out}\left(h + KE + PE\right)_{out} = \Delta\left[m_s\left(u + KE + PE\right)_s\right]$$

For a closed system:

$m_{in} = m_{out} = 0 \quad q = Q/m_s \quad w = W/m_s$

$$q + w = \Delta\left(u + KE + PE\right)$$

For a steady state flow (SSF):

$d(.)/dt = 0;\ \dot{m}_{in} = \dot{m}_{out} = \dot{m} \quad q = \dot{Q}/\dot{m} \quad w = \dot{W}/\dot{m}$

$$q + w = \Delta\left(h + KE + PE\right)$$

To retain of chapter 3 - A

- **Energy is the ability to produce a change, many forms**

Work W, Mechanical power $\dot{W}$:

$$\dot{W} = \mathbf{F} \bullet \mathbf{v} \qquad W = \int_{\vec{\mathbf{x}}_1}^{\vec{\mathbf{x}}_2} \mathbf{F} \bullet d\mathbf{x}$$

3 conditions:

- *Crosses system boundaries,*
- *due to force ON the system,*
- *producing displacement*

Examples of work of point forces:

- ***Power transmission:*** $\text{Power} = T\omega$
- ***Elastic*** $W = \frac{-1}{2} k \left(x_2^2 - x_1^2 \right)$
- ***Electric*** $\text{Power}: -\Sigma_i \mathcal{V}_i I_i$

- **Work depends on the process (the path)**

- **Work of changing volume: if:**
  - **No friction ($\mathbf{S} = P\,\mathbf{n}$)**
  - **Equilibrium (pressure P uniform)**
  - **Closed system ($\mathbf{n}.\mathbf{v}_s = \mathbf{n}.\mathbf{v}_m$)**

$$\left.\right\} W = -\int P dV$$

- **Flow work is:** $P_1 V_1 - P_2 V_2$
- **For open system:** $\int_1^2 V dP$

- **Work of surface tension:** $\int \sigma \, dA$

To retain of chapter 3 - B

Internal energy U:

Energies (kinetic, potential, and chemical) of molecules: State property

- ***Internal energy + Flow work ⇒ Enthalpy***

Heat power $\dot{Q}$:

- **Crosses the boundary**
- **Due to a temperature difference**
- **NOT associated with a transfer of matter**

- ***The general form of the First Law is:***

$$\dot{Q}+\dot{W}+\dot{m}_{in}\left(h+KE+PE\right)_{in}-\dot{m}_{out}\left(h+KE+PE\right)_{out}=\frac{d}{dt}\left[m_s\left(u+KE+PE\right)_s\right]$$

- ***Special case, closed system:***

$$q+w=\Delta\left(u+KE+PE\right)$$

- ***Special case, Steady Flow:***

$$q+w=\Delta\left(h+KE+PE\right)$$

Geothermal Energy Capacity Building in Egypt (GEB)

Thermal Engineering for Geothermal Energy

4 – Pure substance & gas mixtures

Co-funded by the
Erasmus+ Programme
of the European Union

Definitions

Phase

A Group of molecules having:

- **<u>Homogeneous</u> <u>chemical</u> Composition**
- **<u>Homogeneous</u> <u>physical</u> Properties**

Examples

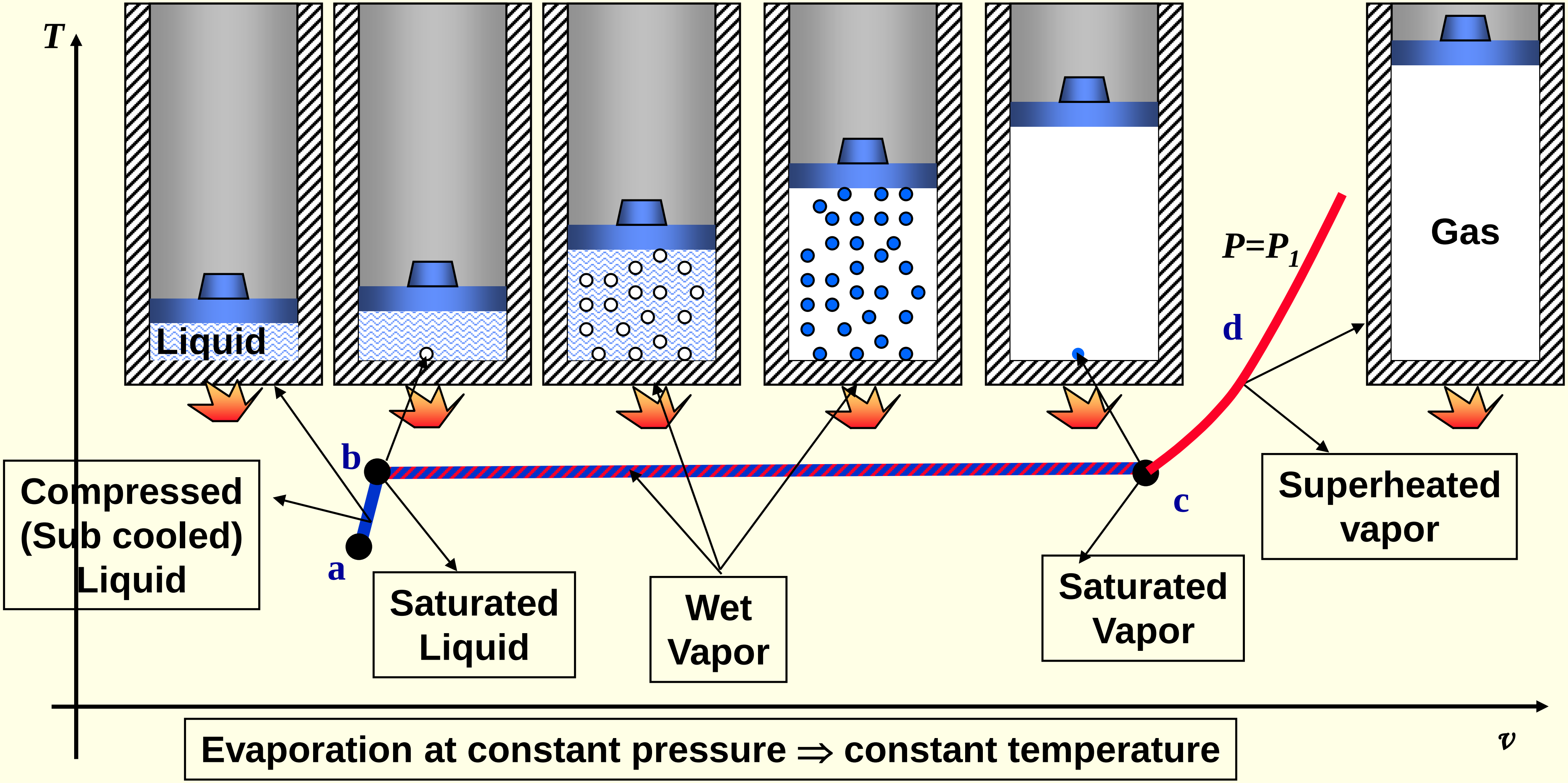
- **Gas**
- **Liquid**
- **Solid**
  - **various crystalline structures**

Pure substance

<u>Homogeneous</u> and <u>invariable</u> chemical Composition in <u>all</u> phases

H_2O, CO_2, …
But air ??

Evaporation at Constant Pressure 1



Compressed (Sub cooled) Liquid

Saturated Liquid

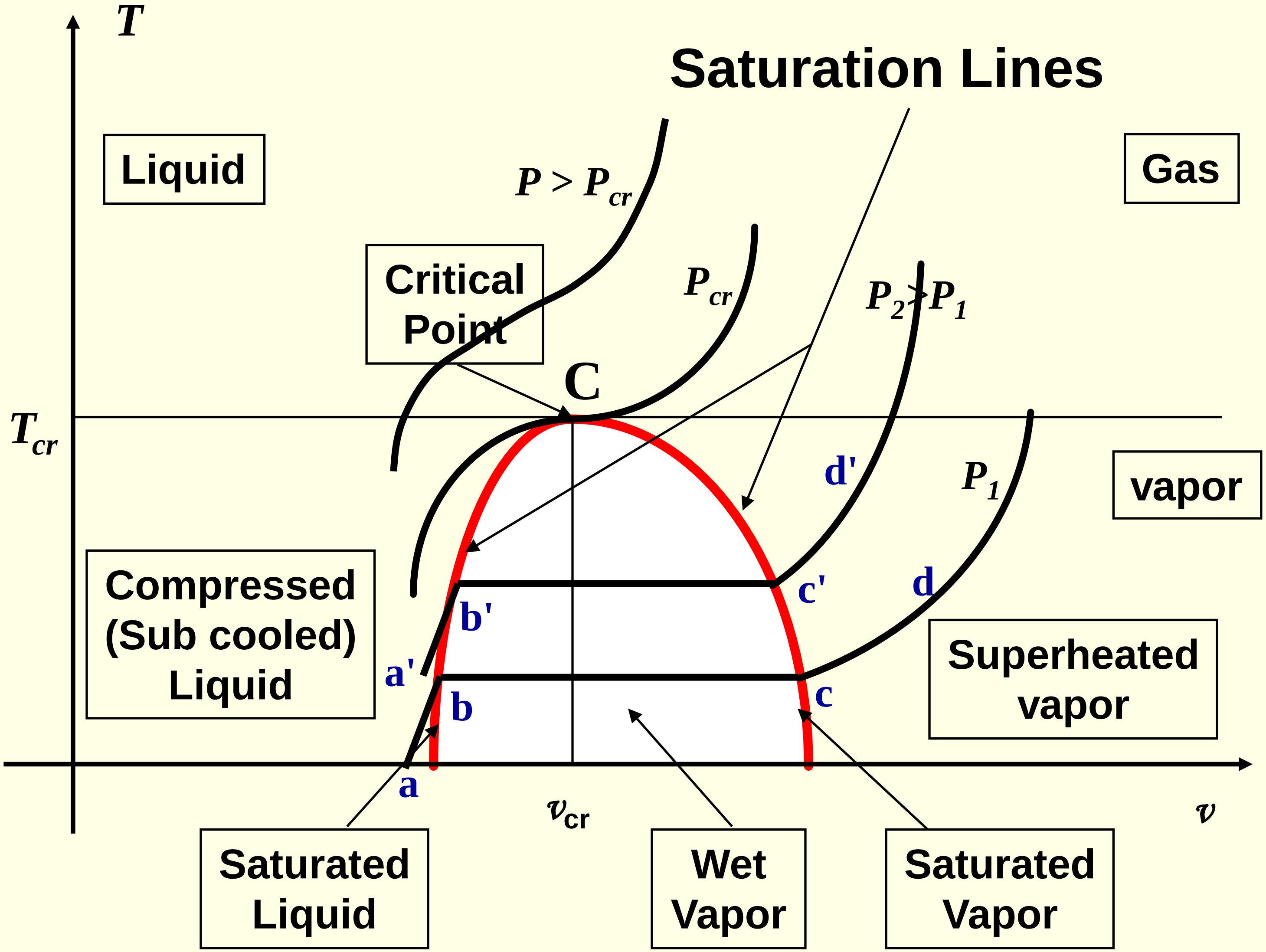
Wet Vapor

Saturated Vapor

Superheated vapor

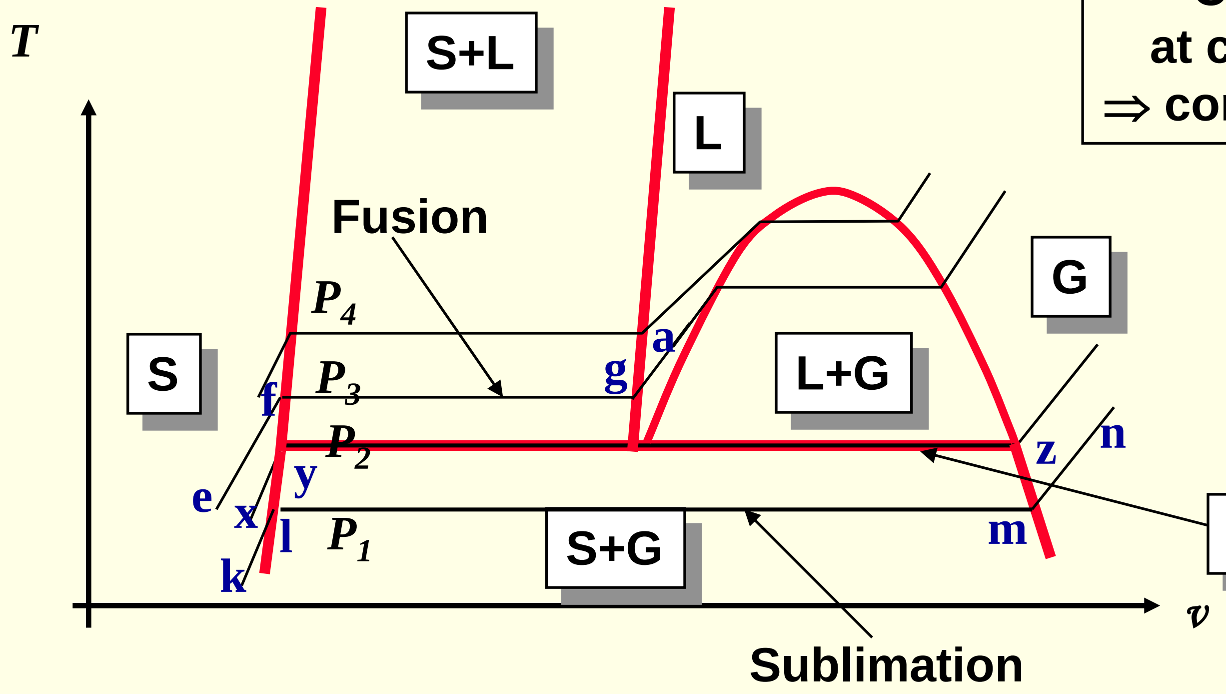
Evaporation at constant pressure ⇒ constant temperature

Evaporation at Constant Pressure 2



Fusion and Sublimation

Changing phase
at constant pressure
⇒ constant temperature



G Gas

L Liquid

S Solid

Triple Line (S+L+G)

If we heat a solid at const P: **⇒ fusion at constant *T*** **⇒ evaporation at constant *T***

The operation can be repeated at other pressures => lines of saturation

If we heat a solid at very low pressure: **⇒ sublimation at constant *T***

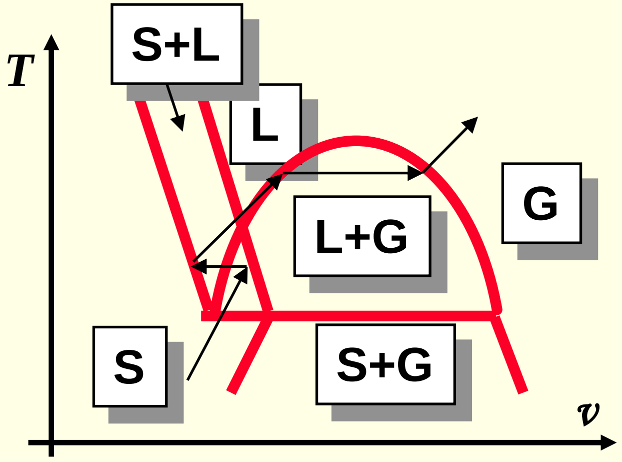
Ice cubes float !?

Water:
A special case

Ice cubes float:

Solid Density < liquid density

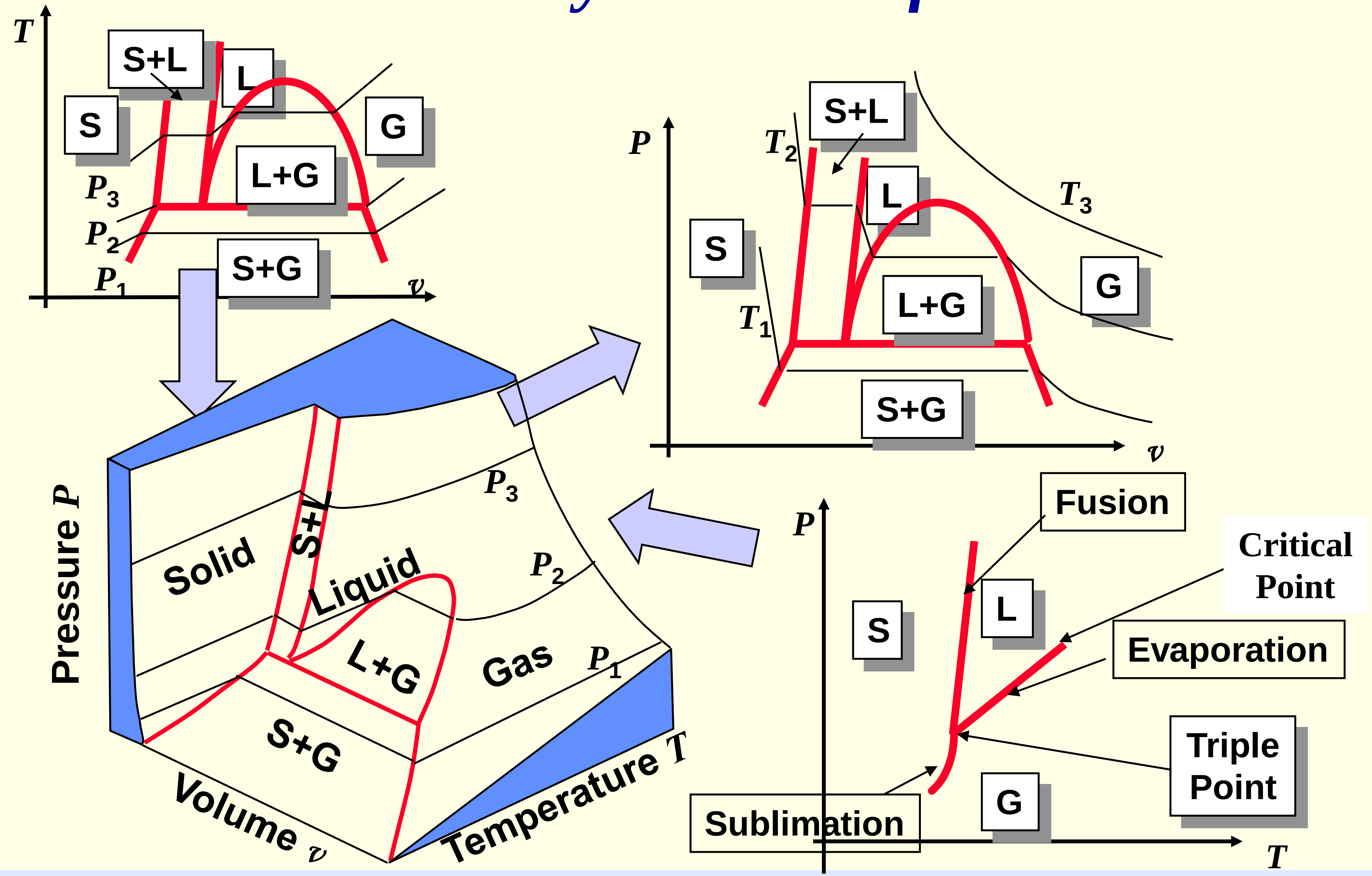
Ice cubes shrink during fusion



Latent Heat of:	**Energy needed to transform 1 kg**	
	From	**To**
Fusion	**Solid**	**Liquid**
Evaporation	**Liquid**	**Vapor**
Sublimation	**Solid**	**Vapor**

Additional

Thermodynamic Surfaces



Thermodynamic Tables

Thermodynamic (pressure or temperature) _saturation_ Tables:

P	T	v_f	v_g
…	…	…	…
…	…	…	…

Thermodynamic _superheat_ Tables:

	P=…	P=…	P=…
T=…	v=…	v=…	v=…
T=…	v=…	v=…	v=…
T=…	v=…	v=…	v=…

For wet vapor :

Dryness Fraction (quality)

$x = m_G / (m_L + m_G)$

$$\begin{aligned} v &= x\, v_g + (1-x)\, v_f \\ &= v_g - (1-x)\, v_{fg} \\ &= v_f + x\, v_{fg} \end{aligned}$$

For a compressed liquid :

If no tables, use:

$v(T,P) \approx v_{satL}(T)$

Online data, ex:

https://webbook.nist.gov/chemistry/fluid/

Degrees of Freedom F

F = **Minimum** Number of state variables necessary to **define** the state

$\mathcal{P}$ = Number of Phases

C = Number of chemical componants

Simple Case 1: Pure substance (C = 1)
Single phase ($\mathcal{P}$ = 1)

In this case: Number of Degrees of Freedom
= Number of different forms of energy exchange:

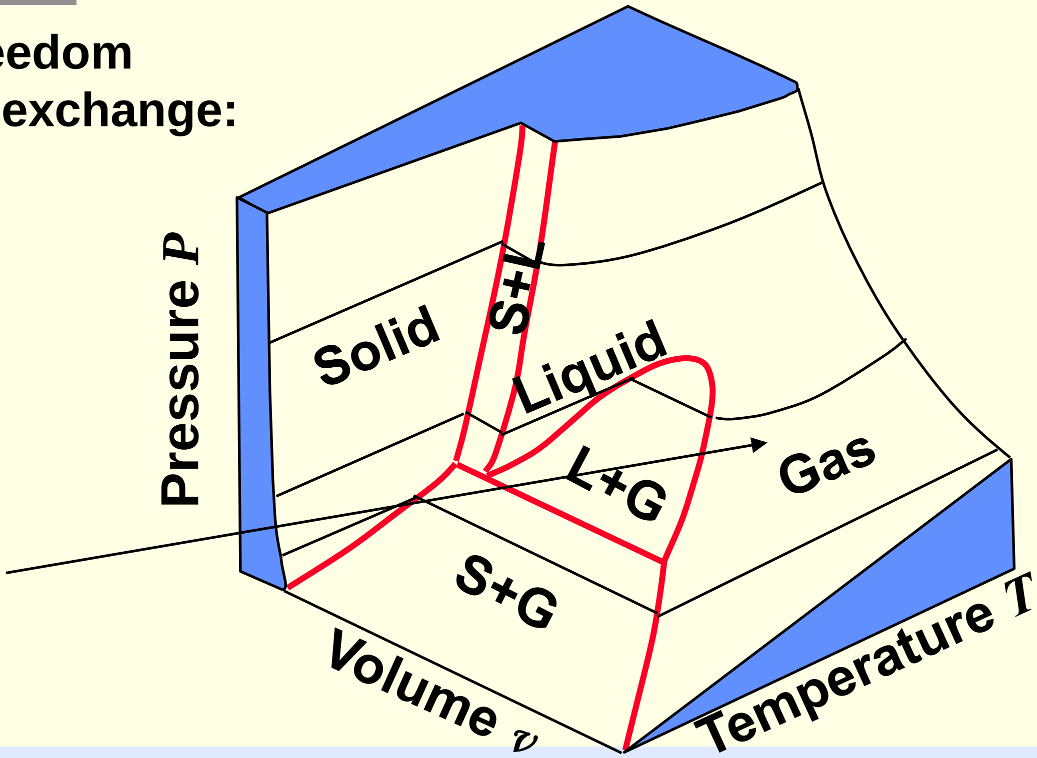
If only:

- Heat
- work of changing volume

⇨ F = 2

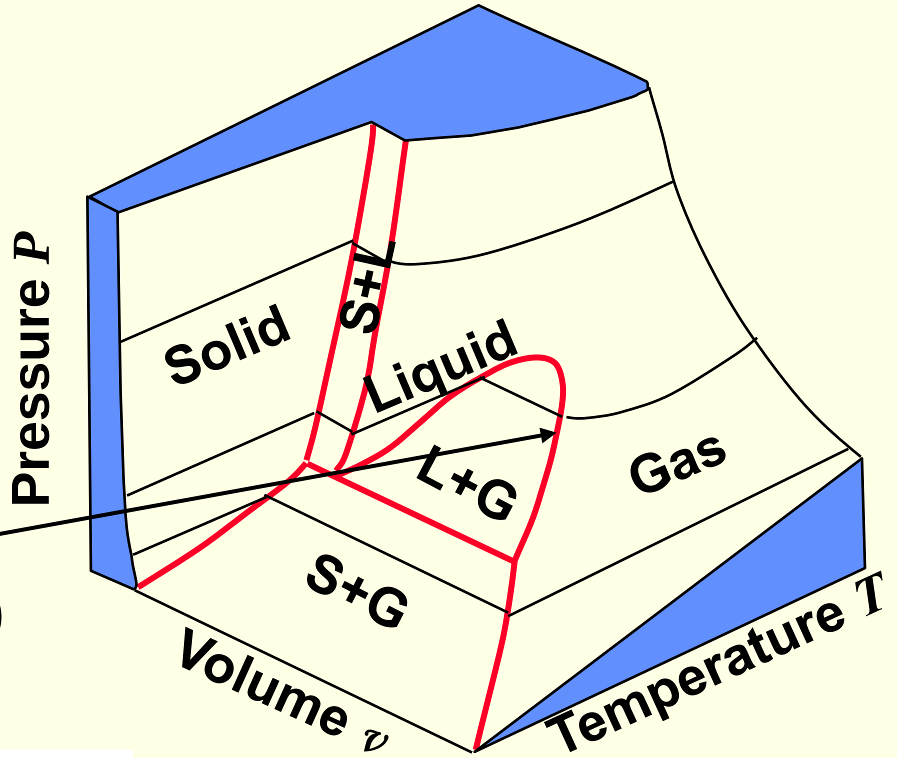
Example :

To localize the state of a gas,
We need to know (P, T), (P, v) or (v, T)



Gibbs Phase Rule

Simple Case 2: Pure substance ($C = 1$)
2 phases in equilibrium ($\mathcal{P} = 2$)



Equilibrium imposes a condition

⇨ $F = 2-1 = 1$

Example **:** **To localise the state**
Of a saturated vapor,
It is sufficient to know (P) or (T)

General Case : If number of components $C > 1$

Every new component adds a new degree of freedom

⇨ $$\boxed{F = C - \mathcal{P} + 2}$$

Gibbs Phase Rule for
Two forms of energy exchange

State Equations

Assume a number F (degrees of Freedom) of state variables are given:

All other state variables can be expressed in terms of them

These relations are: → **State Equations**

Mainly 2 kinds:

- Thermal equation of state : $f(P, T, v) = 0$ $(v = 1/\rho)$
- Calorific equations of state $u = f_1(P, T)$

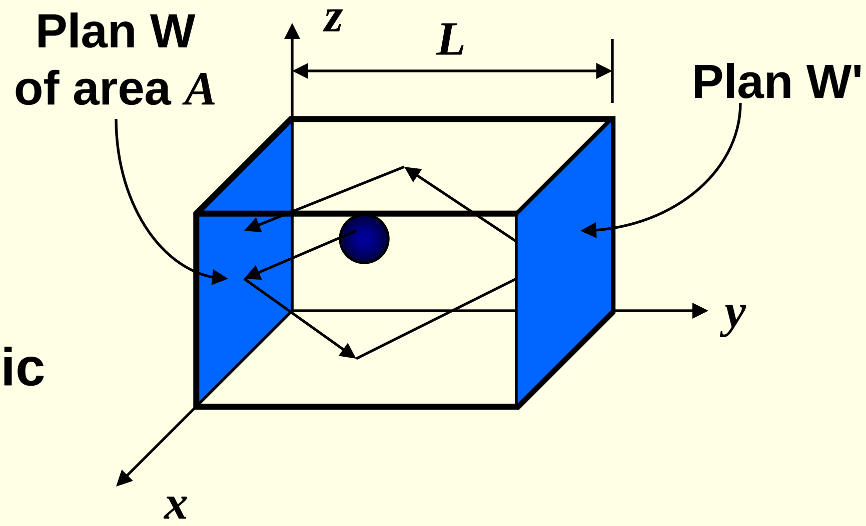
  $h = f_2(P, T)$

Perfect (Ideal) Gas

Hypotheses:

- **Low density**
  - volume of molecules Negligible
  - intermolecular forces Negligible
  - time of impact Negligible
- **Molecular collisions are perfectly elastic**

  (No energy loss during impact)



Change of the momentum of a particle during collision with the plan W (component in y):

$$\boxed{\Delta\,(\textbf{momentum}) = m_0\,(v_y - (-\,v_y)) = 2m_0 v_y}$$

Average Time between 2 collisions on the plan W:

$$\boxed{\Delta t = 2\,L\,/\,v_y}$$

Pressure due to a molecule:

$$\boxed{P_{1\ \text{molecule}} = Force/A = [\Delta\,(momentum)/\Delta t\,]/A = m_0 v_y^2\,/\,V}$$

For *N* molecules:

$$\boxed{P\,V = m_0\,\Sigma\,v_y^2}$$

Equation of State for a perfect gas

Principle of equipartition of energy:

$$\Rightarrow \quad \Sigma\, v_y^2 = \Sigma\, v_x^2 = \Sigma\, v_z^2 = 1/3\, \Sigma\, v^2 = 1/3\, N\, v_{rms}^2$$

$$\boxed{P\,V = m_0\, \Sigma\, v_y^2} \quad \Rightarrow \quad \boxed{P\,V = m_0\, 1/3\, N\, v_{rms}^2}$$

perfect gas ⇒ $T \propto$ average kinetic energy of a molecule $\Rightarrow \quad \boxed{\underset{\textbf{Boltzmann}}{k}\, T = 1/3\, m_0\, v_{rms}^2}$

$$\boxed{P\,V = N\,k\,T = \underset{\textbf{moles}}{n}\, \underset{\textbf{Avogadro}}{N_A}\, k\,T = n\,\bar{R}\,T}$$

$\bar{R} = N_A k =$ **8314.5 J/kmol K**

Universal Gas Constant

Gas Constant: $R = \bar{R} / \mu$

(μ molecular mass)

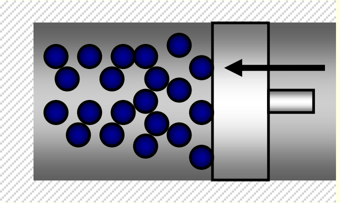
$$\boxed{P\,V = n\,\bar{R}\,T} \quad \Rightarrow \quad \boxed{P\,\bar{v} = \bar{R}\,T}$$

$n = m / \mu$

(m mass of gas) $\Downarrow$

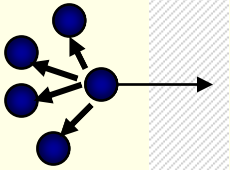
$$\boxed{P\,V = m\,R\,T} \quad \Rightarrow \quad \boxed{P\,v = R\,T}$$

Real gases Equations of state



Van der Waals (VDW):

- Volume of molecules non negligeable
- Intermolecular forces non negligeable



⇒ $(P + a/v^2)(v - b) = RT$ (a, b const.)

a & b are functions of $P_R = P/P_c$; $T_R = T/T_c$;

Law of corresponding states :

Compressibility: $Z = Pv / RT$

For all Gases : $Z = f(P_R, T_R)$
(see compressibility charts)

If gas at low density:
(ex. $P_R < 0.1$ and/or $T_R > 5$) then $Z \approx 1$
i.e. *almost a perfect gas*

!

- The precision of VDW is better than $Pv = RT$; BUT not perfect
- The error of compressibility < 2.5% for all gases

Energy Properties

Internal Energy	**Total Internal Energy**	**Enthalpy**	
U	E	H	**J**
u	e	h	**J/kg**
$\bar{u}$	$\bar{e}$	$\bar{h}$	**J/kmol**

Thermal Capacities
(Specific heats)

Thermal Capacities at constant volume

$$c_v = \partial u / \partial T\,|_v \qquad \bar{c}_v = \partial \bar{u} / \partial T\,|_v = \mu\, c_v$$

Thermal Capacities at constant pressure

$$c_P = \partial h / \partial T\,|_P \qquad \bar{c}_P = \partial \bar{h} / \partial T\,|_P = \mu\, c_P$$

Ratio of thermal capacities: $\gamma = c_P / c_v$

(Specific heats ratio)

Calorific Eq. of State – Perfect Gas

Reminder:

Assuming translational movement only (degrees of freedom $f = 3$)
+ principle of equipartition of energy:

$$k\,T = 1/3\; m_0\, \mathrm{v}_{rms}^2$$

In general, if $f \neq 3$:

$$f\,k\,T/2 = 1/2\; m_0\, \mathrm{v}_{rms}^2 = KE \text{ for one molecule}$$

For a perfect gas, u is composed of KE only (no intermolecular forces)

No. of molecules in 1 kg: N_A / μ

⇒ $$u = (N_A\, k/\mu)\,(f/2)T = R\,(f/2)\,T$$

Also: $$h = u + P v = R\,(1 + f/2)\,T$$

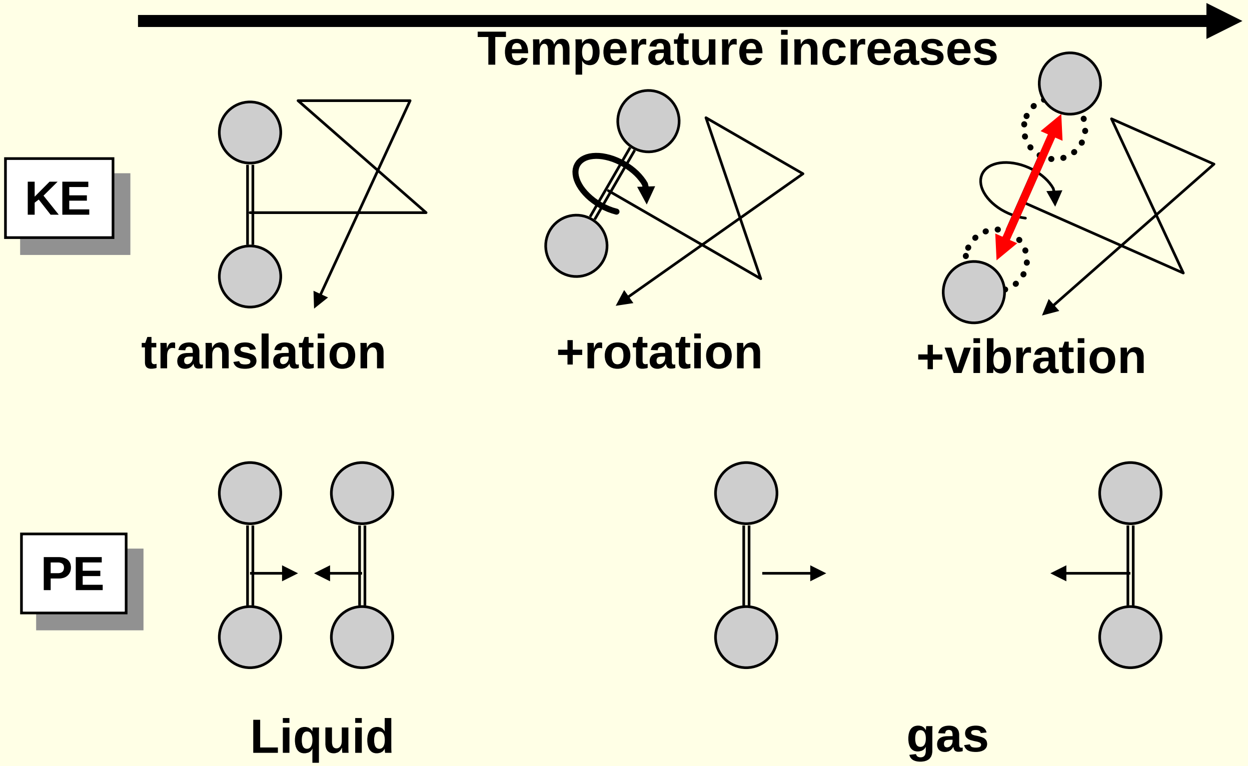
⇒ $$c_v = \partial u/\partial T|_{v=const} = R\,(f/2)$$
$$c_P = \partial h/\partial T|_{P=const} = R\,(1+f/2)$$

$$\gamma = c_P/c_v = (f+2)/f$$

$$c_P - c_v = R$$

⇒ $$\Delta u = c_v\,\Delta T; \qquad c_v = R/(\gamma - 1)$$
$$\Delta h = c_P\,\Delta T; \qquad c_P = \gamma R/(\gamma - 1)$$

Variation of Internal Energy with temperature (qualitative)



Temperature increases

KE

translation

+rotation

+vibration

PE

Liquid

gas

+ Dissociation
+ Ionization + ...

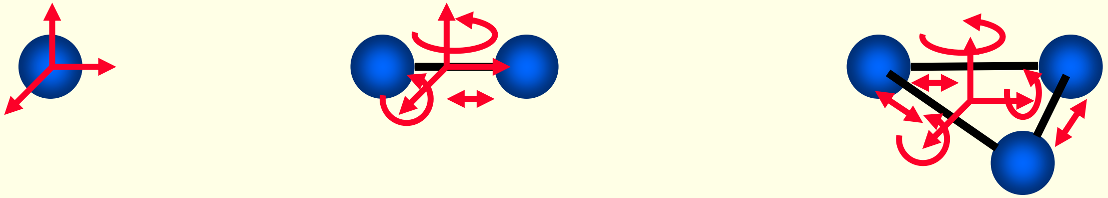
Gas, large variations of temperature 1

To estimate thermal capacities of a perfect gas:

$$f \Rightarrow \gamma = (f+2)/f \Rightarrow c_v = R/(\gamma-1) \Rightarrow c_P = \gamma c_v$$

But f depends on temperature T and the nature of molecules

To estimate f:



Temperature	**Movement**	**Mono atomic** (Argon, Neon, …)	**Diatomic** (O_2, N_2, …)	**Poly atomic** (CO_2, NH_3, …)
Low	**Translation**	$f = 3$	$f = 3$	$f = 3$
Medium	**+Rotation**	$+0$	$+2$	$+3$
High	**+Vibration**	$+0$	$+1$	$+n?$

Gas, large variations of temperature 2

If

- **Gas**
- **Very small density**
- **Large variation of *T***

Then:

$Pv = RT$

$c_P, c_v = f(T)$

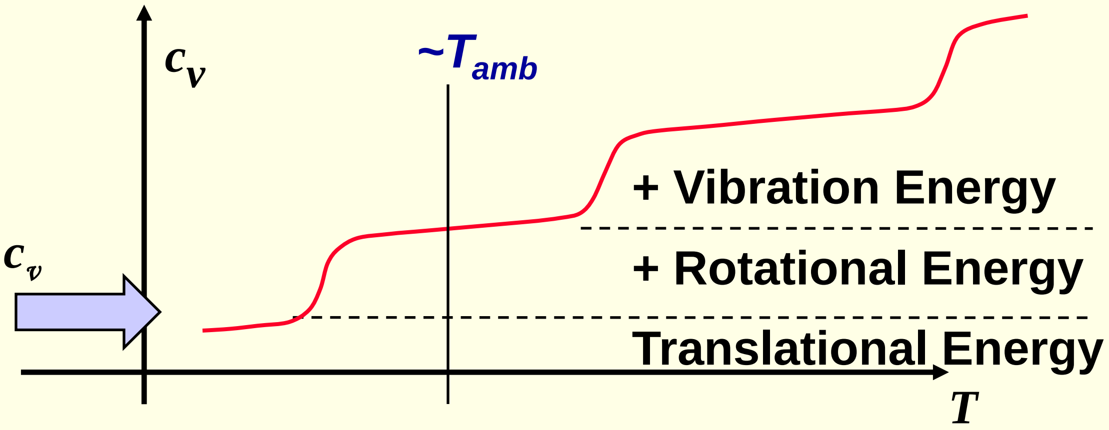
$u, h = f(T)$

are not $f(P)$

Semi-ideal gas

Every degree of freedom is activated at a given temperature.

Typical variation of c_v At low pressure:



$$c_v = \partial u/\partial T|_v = du / dT$$

$$c_P = \partial h/\partial T|_P = dh / dT = d(u+Pv)/dT = d(u+RT)/dT = c_v + R$$

$c_P = f(T)$ at low pressure is given in tables, hence:

$$\boxed{h = \int c_P\, dT; \qquad c_v = c_P - R; \qquad u = \int c_v\, dT}$$

Calorific Eq. of state – Other phases

For a mixture liquid-vapor:

$$u = x\, u_g + (1\text{-}x)\, u_f$$
$$h = x\, h_g + (1\text{-}x)\, h_f$$

Latent heat :

$$h_{fg} = h_g - h_f$$
$$c_P \rightarrow \infty$$

For a solid or liquid:

$$c_P = c_v = c$$

For crystals
at ambient temperature:

$$c = 3\, \bar{R} / \mu$$

Ideal Gas Mixtures

Hypothesis

Very Low Density

⇨ Negligible Molecular Interactions

⇨ Each component acts as an independent ideal gas

Definitions

Mass Fraction

mass of componant *i*

$\boxed{x_i = m_i / m_T}$ $\boxed{m_T = \Sigma_i m_i}$ $\Sigma_i x_i = 1$

total mass

Molal Fraction

number of moles of componant *i*

$\boxed{y_i = n_i / n_T}$ $\boxed{n_T = \Sigma_i n_i}$ $\Sigma_i y_i = 1$

total number of moles

Mixture Molecular Mass

One can define an « equivalent » molecular mass:

$$\boxed{\mu_{mix} = m_T / n_T = \Sigma\, m_i / n_T = \Sigma\, n_i\, \mu_i / n_T = \Sigma\, y_i\, \mu_i}$$

Also:

$$\boxed{\mu_{mix} = m_T / n_T = m_T / \Sigma\, n_i = m_T / (\Sigma\, m_i / \mu_i) = 1 / (\Sigma\, x_i / \mu_i)}$$

Conversion Rules:

$$\boxed{x_i = m_i / m_T = n_i\, \mu_i / m_T = (n_i/n_T)\, \mu_i / (m_T/n_T) = y_i\, \mu_i / \mu_{mix}}$$

$$\boxed{y_i = x_i\, \mu_{mix} / \mu_i}$$

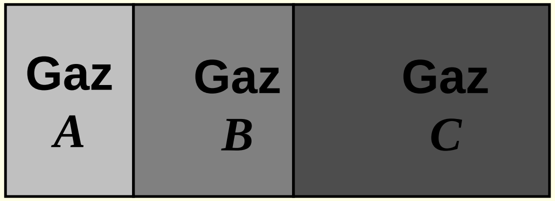
Additional

Amagat Model

Every component *i* behaves as a separate perfect gas

Having the following state properties:

- $P = P_{mix}$, $T = T_{mix}$
- V_i **is the ‘partial volume’ :** $V_i < V_{mix}$



$$P\,V_i = n_i\,\bar{R}\,T \qquad (*)$$

Summing for all components:

$$P\,\Sigma\,V_i = n_T\,\bar{R}\,T \qquad (\#)$$

Comparing with $PV = n\bar{R}T$ **gives**

$$\Sigma\,V_i = V_{mix}$$

Dividing equation (*) by (#)

$$V_i\,/\,V = n_i\,/\,n_T = y_i$$

Dalton Model

Every component *i* behaves as a separate perfect gas

Having the following state properties:

- $V = V_{mix}, T = T_{mix}$
- P_i **is the 'partial pressure' :** $P_i < P_{mix}$

$$P_i\, V = n_i\, \bar{R}\, T \qquad (*)$$

Summing for all components

$$\Sigma\, P_i\, V = n_T\, R\, T \qquad (\#)$$

Comparing with $PV = n\bar{R}T$

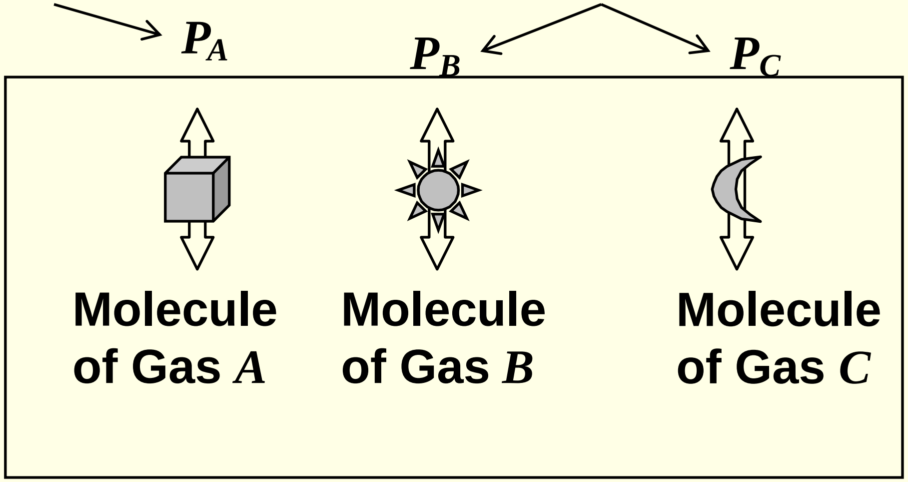
$$\Sigma\, P_i = P_{mix}$$

Dividing equation (*) by (#)

$$P_i / P = n_i / n_T = y_i$$

Pressure due to impact Of molecules of gas A

Pressure due to B and C molecules



P_A P_B P_C

Molecule of Gas *A*

Molecule of Gas *B*

Molecule of Gas *C*

Energy properties of Ideal gas mixtures

$$u_{mix}(T) = \sum_i x_i\, u_i(T) \qquad h_{mix}(T) = \sum_i x_i\, h_i(T)$$

$$c_{vmix}(T) = \sum_i x_i\, c_{vi}(T) \qquad c_{Pmix}(T) = \sum_i x_i\, c_{Pi}(T)$$

$$\bar{u}_{mix}(T) = \sum_i y_i\, \bar{u}_i(T) \qquad \bar{h}_{mix}(T) = \sum_i y_i\, \bar{h}_i(T)$$

$$\bar{c}_{vmix}(T) = \sum_i y_i\, \bar{c}_{vi}(T) \qquad \bar{c}_{Pmix}(T) = \sum_i y_i\, \bar{c}_{Pi}(T)$$

Special mixture: Air – Water vapor

Psychrometry

Problem considered

A perfect mixture of perfect gases:

- Each component : Perfect Gas
- A small quantity of matter may change phase
  (Gas – Liquid or Gas – Solid)

Simplified Model

- gaseous phase is homogeneous and composed of 2 perfect gases
- vapor is at low pressure

⇨ •Treated is a perfect gas, even when close to saturation

- Liquid and solid phases are void of dissolved gases

Definitions

Absolute (Specific) Humidity

$$\omega = m_{vap} / m_a$$

- m_{vap} mass of vapour
- m_a mass of other gases

$$\omega = (P_{vap}\, V / R_{vap}\, T) / (P_a\, V / R_a\, T) = (\mu_{vap} / \mu_a)\,(P_{vap} / P_a)$$

(For air and water: $\omega = 0.622\, P_{vap} / P_{air}$)

Relative Humidity

$$\phi = P_{vap} / P_{vap\_sat}$$

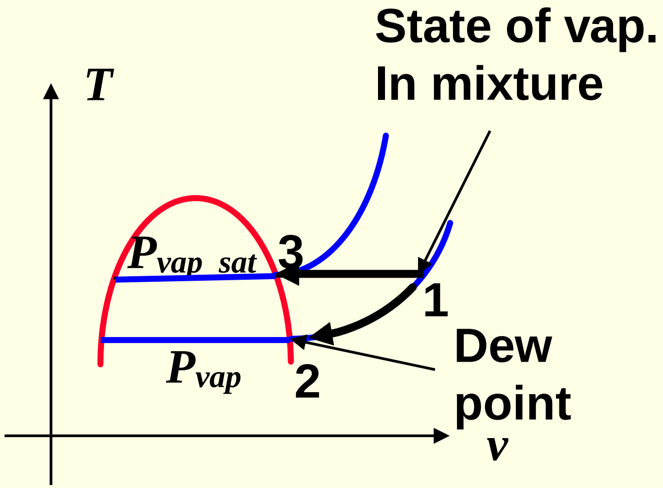
P_{vap_sat} (T) Saturation pressure At mixture temperature

Max Quantity of vapor when: $\phi = 100\%$

Starting from state 1, we can :

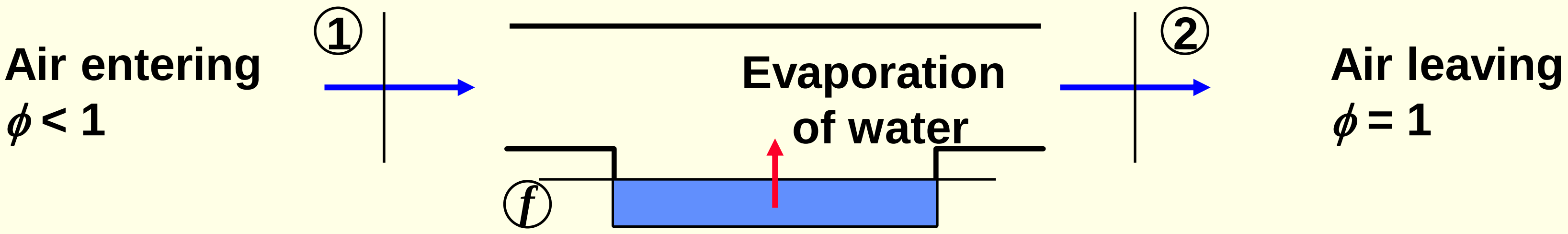
- Cool at const P until state 2 (ϕ reaches 100%)
- Add vapor at T const. Until state 3 (ϕ reaches 100%) (beyond points 2 or 3: condensation)

(For air and water: $\phi = \omega\, P_{air} / (0.622\, P_{vap\_sat}\,(T))$)



$T - v$ Diagram for Condensable material

Adiabatic Saturation



Air entering
$\phi < 1$

①

Evaporation
of water

②

Air leaving
$\phi = 1$

ⓕ

1st Law

$$\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1} + \dot{m}_w h_{f2} = \dot{m}_a h_{a2} + \dot{m}_{v2} h_{vsat2}$$

But $\dot{m}_w = \dot{m}_{v2} - \dot{m}_{v1}$

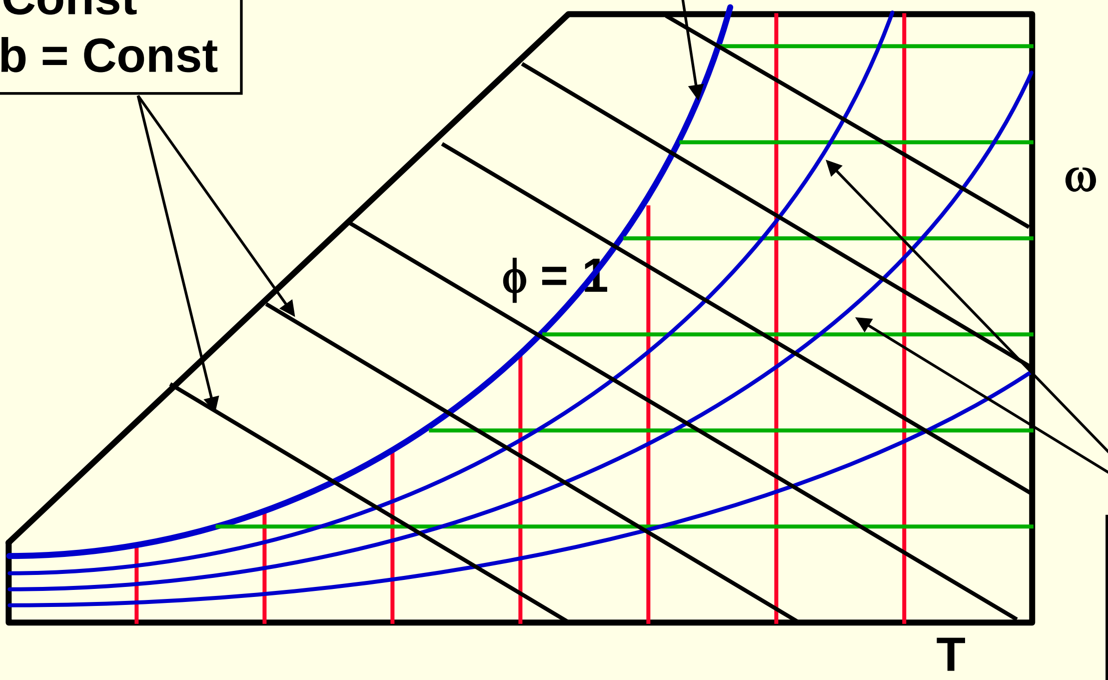
⇨ $h_{a1} + \omega_1 (h_{v1} - h_{f2}) = h_{a2} + \omega_2 h_{fg2}$

T_1 : Temperature of state 1 (dry bulb)

T_2 : Temperature of state 2 (wet bulb)

T wet bulb ≤ T dry bulb

Psychometric Chart



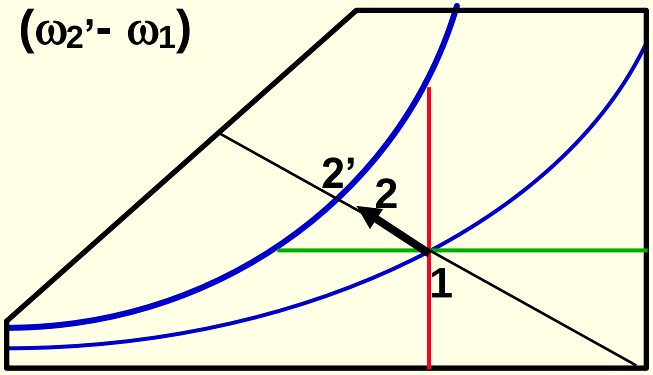
Obtained from: $\phi = 1 = \omega P_{air} / (0.622\, P_{vap\_sat}(T))$

h Tot. = Const
wet bulb = Const

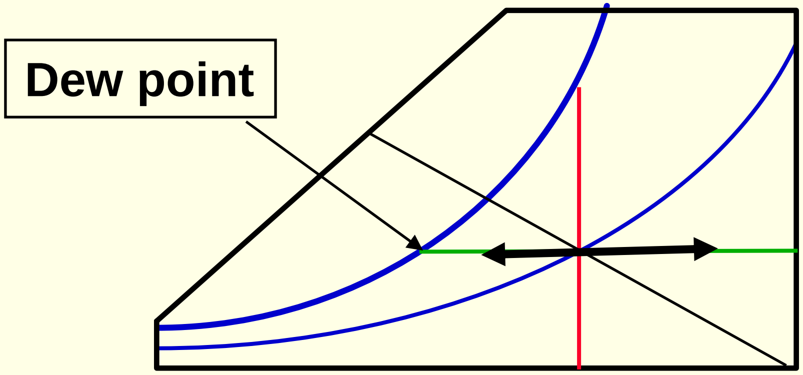
ϕ = Const < 1
(obtained by dividing Ordinates proportionally)

Air conditionning processes

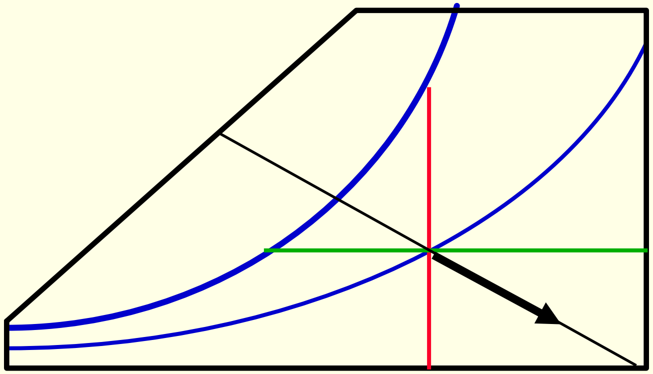
$$\eta = \frac{(\omega_2 - \omega_1)}{(\omega_2' - \omega_1)}$$



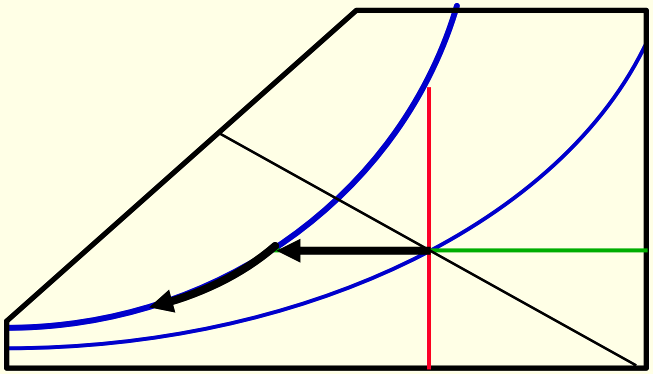
Humidification



Cooling – Heating



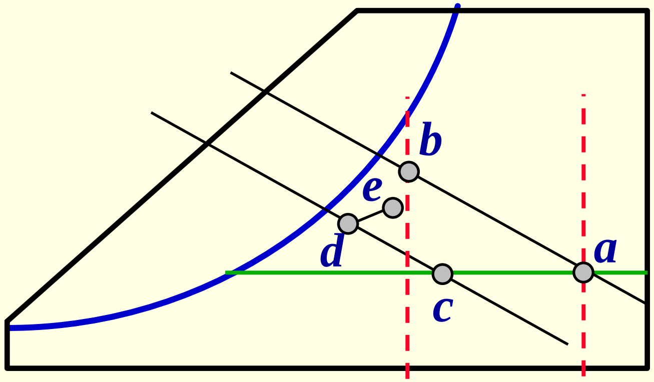
Drying

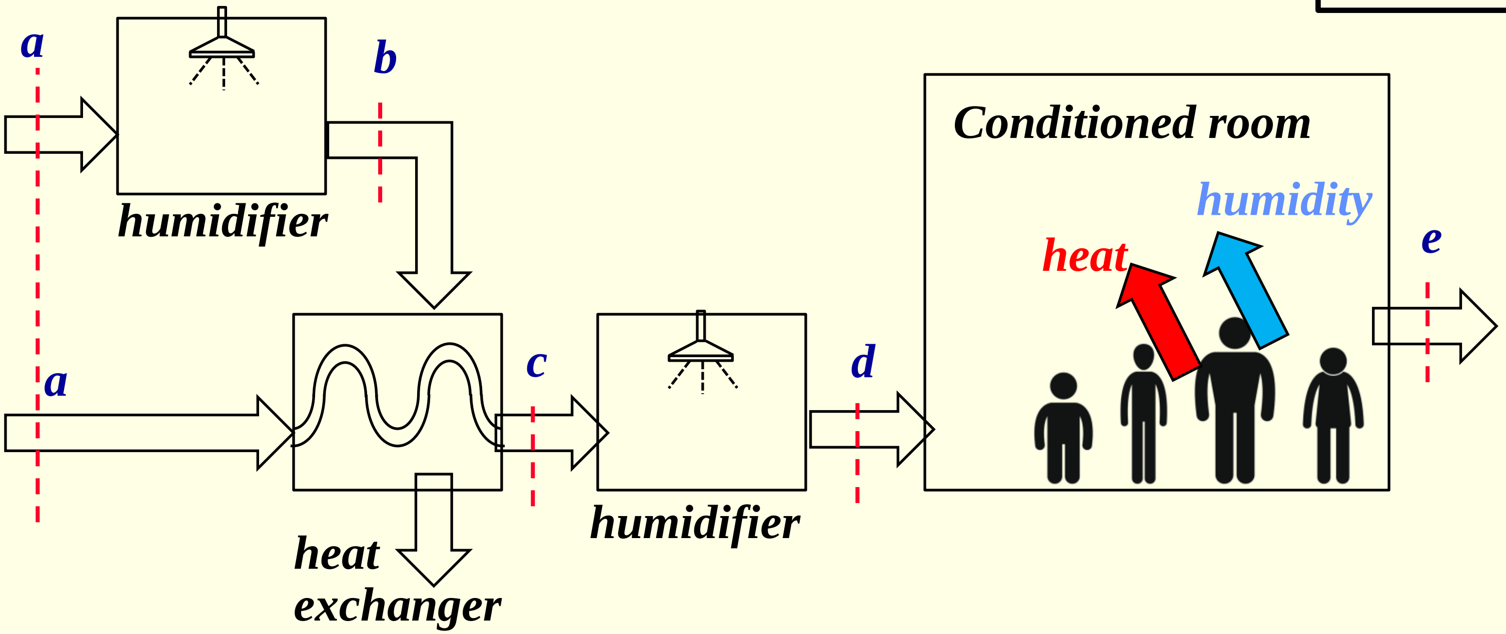


Cooling and Dehumidification

Application: humidification air conditioner

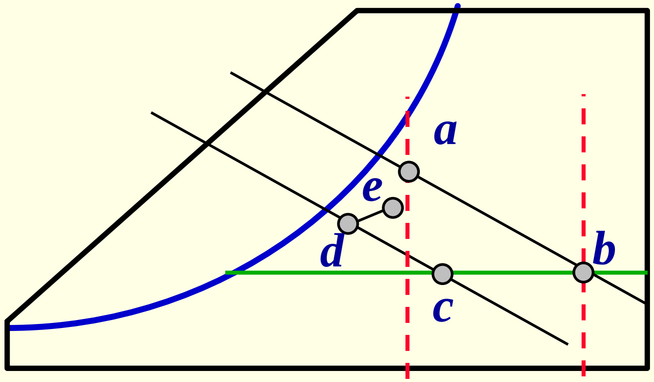
2-stage humidification air conditioner

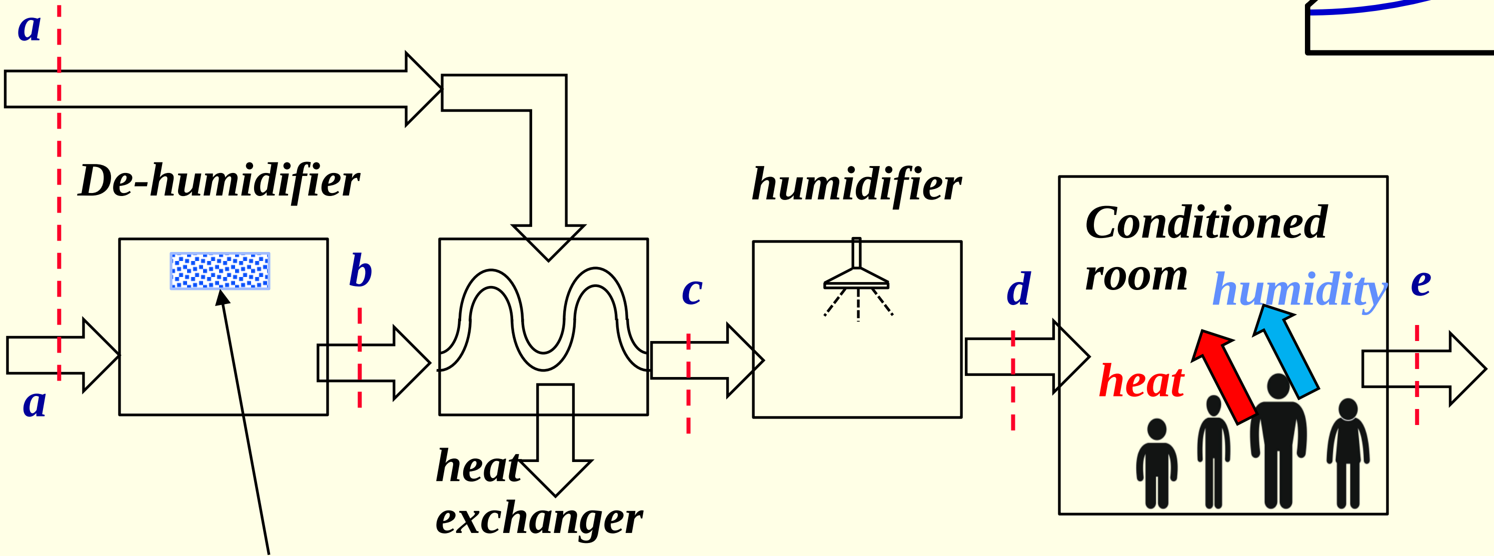




Application: desiccant air conditioner

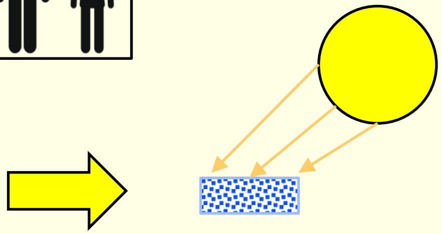
Desiccant based air conditioner





Desiccant

When desiccant becomes wet:



Drying using solar energy

To retain of chapter 4. A

Phase change

- During phase change (evaporation, fusion, …) P = const ⇔ T = Const.
- Latent Heat: heat for complete phase change @ P = const.
- Saturation lines indicate phase change is about to start or end.
- Critical point: vapor and liquid are indistinguishable. Triple line 3 phases in equilibrium
- Dryness fraction (quality) x is percentage of evaporation

Ideal (perfect) gas

- ANY gas at very low density is an ideal (perfect) gas
- Thermal equation of state: $Pv = RT$
- R = gas constant = $\bar{R} / \mu$; $\bar{R} = 8314.5$ J/kmol K, μ is the molecular mass

Calorific equations of state

- For ideal gases small ΔT: $\Delta u = c_v \Delta T$; $\Delta h = c_P \Delta T$; c_p & c_v constants; $c_P - c_v = R$; $c_P / c_v = \gamma$
- For ideal gases large ΔT: $\Delta u = \int c_v dT$; $\Delta h = \int c_P dT$; c_p & c_v $= f(T)$; $c_P - c_v = R$; (semi-ideal)
- For real gases (non-ideal): $u, h = f(T, P)$

To retain of chapter 4. B

Real gases

- ANY gas at HIGH density is NOT ideal, it is called a REAL gas: $Pv \neq RT$
- Many different equations of state, all depend on reduced properties $P_R=P/P_C$, $T_R=T/T_C$
- Use Van Der Waals, Compressibility charts, …

Ideal gas mixtures

- If all gases in the mix are low density: each gas i is ideal, $V_i=V_{mix}$, $T_i=T_{mix}$, $P_i=y_i P_{mix}$
- y_i=mole fraction, x_i=mass fraction; $\mu_{mix}=y_i\ \mu_i / x_i$
- $u_{mix} = \Sigma_i\ x_i\ u_i$; $h_{mix} = \Sigma_i\ x_i\ h_i$; same for c_v, c_P

Psychrometry

- Absolute humidity $\omega=m_{vap}/m_{dry\_air}$; relative humidity $\phi=P_{vap}/P_{vap\_sat}(T)$
- Adding water: $\omega\uparrow$, $T\downarrow$, $\phi\uparrow$: maximum when ϕ=100% $\Rightarrow$ T=wet bulb temperature
- Cooling: ω=const, $T\downarrow$, $\phi\uparrow$: maximum when ϕ=100% $\Rightarrow$ T=dew point

GEB

Geothermal Energy Capacity Building in Egypt (GEB)

Thermal Engineering for Geothermal Energy

5 – Applications of First Law

Co-funded by the
Erasmus+ Programme
of the European Union

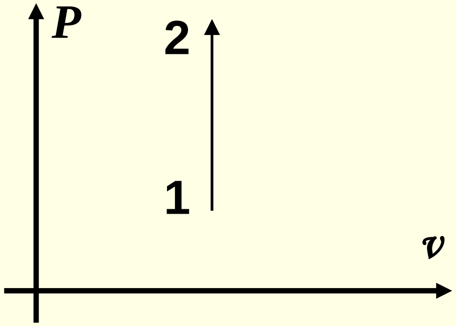
Control Mass Processes

If the only energies involved are:

Heat q, internal energy u
& work of changing volume $w = -\int P\,dv$

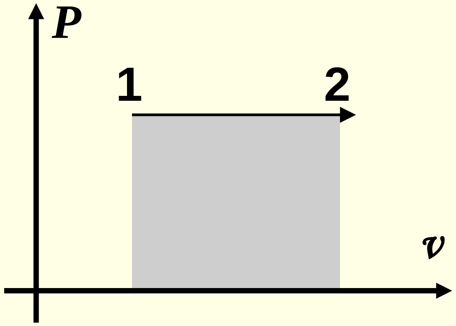
$$q + w = \Delta u$$

Isochoric process
(v = Constant)



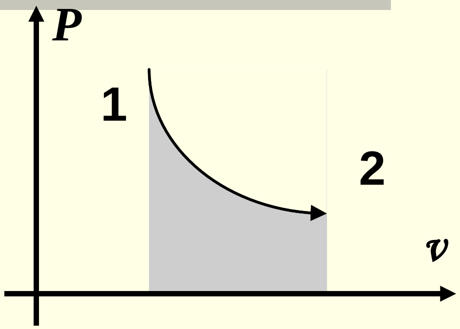
$$w = 0$$
$$q = \Delta u$$

Isobaric Process
(P = Constant)



$$w = -P\,\Delta v$$
$$q = P\,\Delta v + \Delta u \;(= \Delta h\;??)$$

Isothermal process
(T = Constant)



If $u=f(T)$: $q + w = 0$
If perfect gas: $Pv = RT_1$ = Const.

$$w = -\int_{v_1}^{v_2} P\,dv \quad w = -\int_{v_1}^{v_2} \frac{RT_1}{v}\,dv$$

$$w = -RT_1 \ln\left(\frac{v_2}{v_1}\right)$$

Adiabatic process

Adiabatic process (q = 0)

If the only energies involved are:

Internal energy u

&work of changing volume $w = -\int P\,dv$

(no friction)

If in addition perfect gas:

$$w = u_2 - u_1$$

⇨ $-P\,dv = du$

$-P\,dv = c_v\,dT$

Dividing by $P\,v = R\,T$:

⇨ $-dv/v = (c_v/R)\,dT/T$

By Integration
(moderate variation of T
Hence c_v is constant):

$$\ln(T_2/T_1) = (R/c_v)\ln(v_1/v_2)$$

$$T_2/T_1 = (v_1/v_2)^{(R/c_v)}$$

But for a perfect gas : $c_P - c_v = R;\ \gamma = c_P/c_v$

⇨ $R/c_v = \gamma - 1$ $\boxed{T_2/T_1 = (v_1/v_2)^{(\gamma-1)}}$ **1**

By substituting $v = R\,T/P$

⇨ $\boxed{T_2/T_1 = (P_2/P_1)^{(\gamma-1)/\gamma}}$ **2**

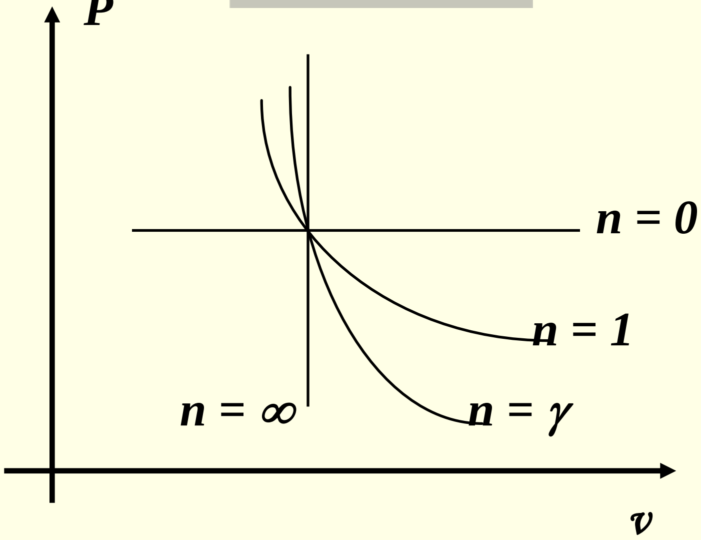
Comparing 1 et 2 $\boxed{P_1\,v_1^{\gamma} = P_2\,v_2^{\gamma}}$

$\boxed{Pv^{\gamma} = const.}$

This is NOT a state equation!
This is a process relation,
i.e. relation between states 1 and 2

Polytropic process

$P\,v^{n} = Const$



Special cases

Isobaric:	$n = 0$
Isothermal:	$n = 1$
Adiabatic:	$n = \gamma$
Isochoric:	$n = \infty$

|Slope adiabatic| > |Slope Isothermal|

$$w = -\int_{v_1}^{v_2} P\,dv \qquad w = -\int_{v_1}^{v_2} \frac{P_1 v_1^{n}}{v^{n}}\,dv$$

$$w = \frac{P_1 v_1^{n}}{n-1}\left(\frac{1}{v_2^{n-1}} - \frac{1}{v_1^{n-1}}\right) \qquad w = \frac{\left(P_2 v_2 - P_1 v_1\right)}{n-1}$$

$n \neq 1$

If perfect gas, then:

$$w = \frac{R\left(T_2 - T_1\right)}{n-1}$$

N.B.:
1- This is a process NOT an equation of state!
2- This is NOT the most general process

Steady State Steady Flow (SSSF) Case I: Flow in ducts: $\dot{Q} = \dot{W} = 0$

Conditions for SSSF:

- $d/dt\,(\text{state variables}) = 0$
- $\Sigma \dot{m}_{in} = \Sigma \dot{m}_{out}$

$$\boxed{\dot{m} = \rho \mathrm{v} A}$$

Adiabatic Flow ($q=w=0$)

$$\Delta h + \Delta KE + \Delta PE = 0$$

A - Liquid

$$\Delta u + \Delta (Pv) + \Delta KE + \Delta PE = 0$$

Negligible Friction : $T \approx$ const $\Rightarrow \Delta u \approx 0$
but $v = 1 / \rho$ = const

$$\boxed{\Delta P/\rho + \Delta \mathrm{v}^2/2 + \Delta g z = 0}$$

Bernoulli !

If friction losses were not negligible: $\Delta u > 0$

$$\boxed{\Delta P/\rho + \Delta \mathrm{v}^2/2 + \Delta g z = -\Delta u}$$

B - Gas

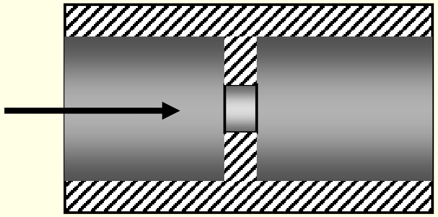
$$\boxed{\Delta h + \Delta \mathrm{v}^2/2 = 0}$$

Valid with or without friction

($\Delta PE \approx 0$)

Work needed to lift a gas $\approx$ 0

Case I (continued):Throttling



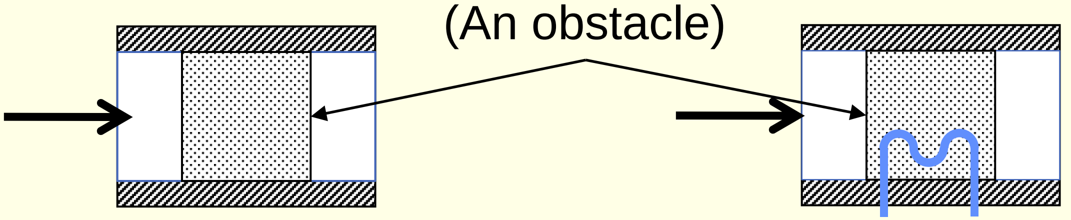
Usually: $w\,,\, q,\, \Delta KE\,,\, \Delta PE \approx 0$

⇒ $\boxed{h_2 = h_1}$

(NB: For a gas, if ΔP was high:
$\Delta\,\rho$ will also be high, Hence: $\Delta\,\mathrm{v},\ \Delta\,KE \neq 0$)

For a perfect gas $h_2 = h_1 \Rightarrow T_2 = T_1$ **[N.B.: $P_2 < P_1$ but $h = f(T)$]**

For a real gas $h = f(T,P)$ Hence for constant h , T varies with P



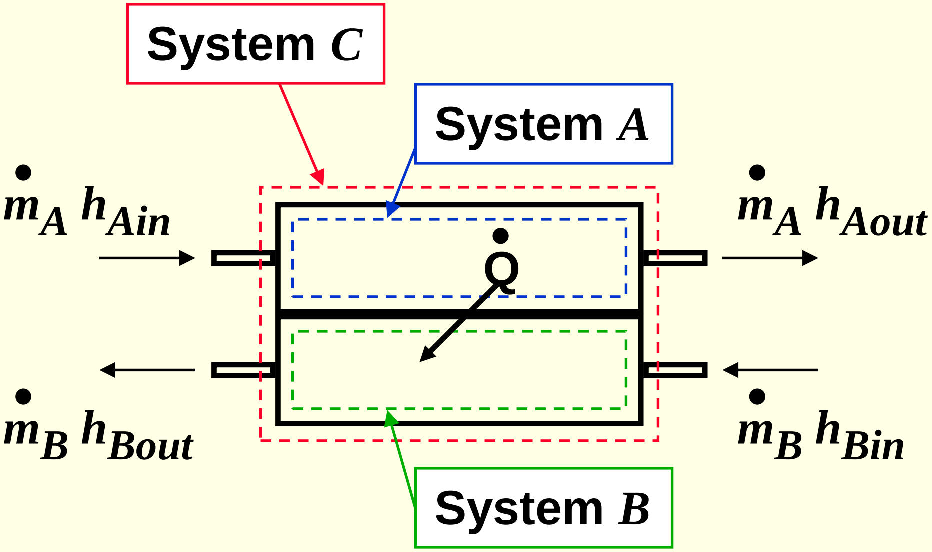
Adiabatic throttling:
$\mu = (\partial T/\partial P)|_h$

Throttling at const. T:
$\mu_T = (\partial h/\partial P)|_T$

For an ideal gas:
$\mu = \mu_T = 0$

Joule-Thomson coefficients:

SSSF: Case II: Heat exchangers: $\dot{Q} \neq 0$; $\dot{W} = 0$



Usually, we neglect:

- **Pumping and/or mixing work,**
- **KE & PE,**
- **Heat lost to surroundings**

Compared to heat exchanged

System *A*: $\dot{Q}_A = \dot{m}_A (h_{Aout} - h_{Ain})$

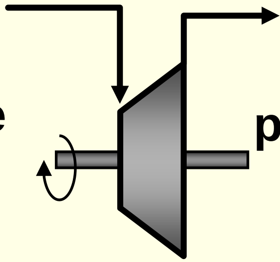
System *B*: $\dot{Q}_B = \dot{m}_B (h_{Bout} - h_{Bin})$

System *C*: $\dot{m}_A (h_{Aout} - h_{Ain}) = - \dot{m}_B (h_{Bout} - h_{Bin})$

$\dot{Q}_A = - \dot{Q}_B$

SSSF: Case III: Fluid machines: $\dot{W} \neq 0$

Work < 0



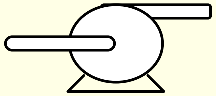
Fluid entering at high pressure

Fluid leaving at low pressure

Turbine

Work > 0

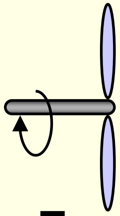
Liquid



Pump

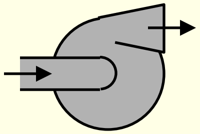
Gas

Classification according to ΔP



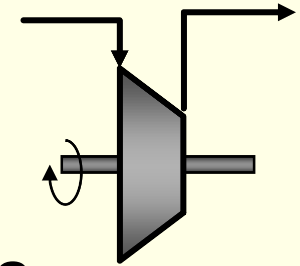
Fan

$P_2 \approx P_1$



Blower

$P_2 > P_1$



Compressor

$P_2 >> P_1$

Usually we neglect:
- Heat
- KE + PE

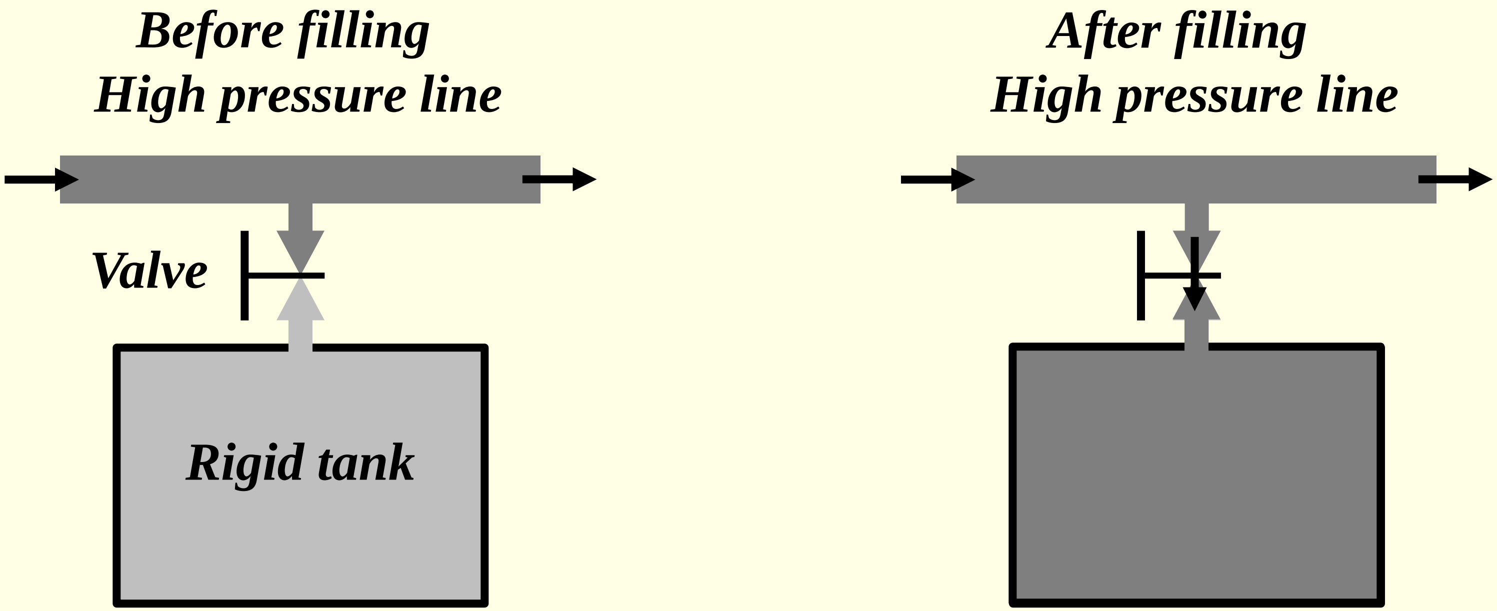
wrt work

$$\Rightarrow \quad \boxed{\dot{W} / \dot{m} = w = h_2 - h_1}$$

Except for:
- Cooled Compressor
- KE for a fan

$$q + w = h_2 - h_1$$
$$w = h_2 - h_1 + \Delta KE$$

Non-Steady Flow: Filling a tank



Iff rigid tank: V is const., W = 0

Energy balance: $Q - 0 + m_{in}(h_{line} + \frac{1}{2}\,\mathrm{v}^2) - 0 = (m\,u)|_{after} - (m\,u)|_{before}$

Mass balance: $m_{in} = m_{after} - m_{before}$

Rigid tank: $(m\,v)|_{after} = V = (m\,v)|_{before}$

$\Rightarrow$ *Solve the problem!*

+ model: thermal eq. of state + calorific eq. of state

The 7 Golden questions

To "understand" any problem, ⇒ ask ***"The 7 Golden questions"***

Group 1

System(s)?

One or more …

Conservation(s)?

Mass, momentum, charge, energy

Energies?

Heat, Work, h, u, KE …

Group 2

Initial, final state(s)?

Given one or more property?

Process(es)?

Constant T, P, v …

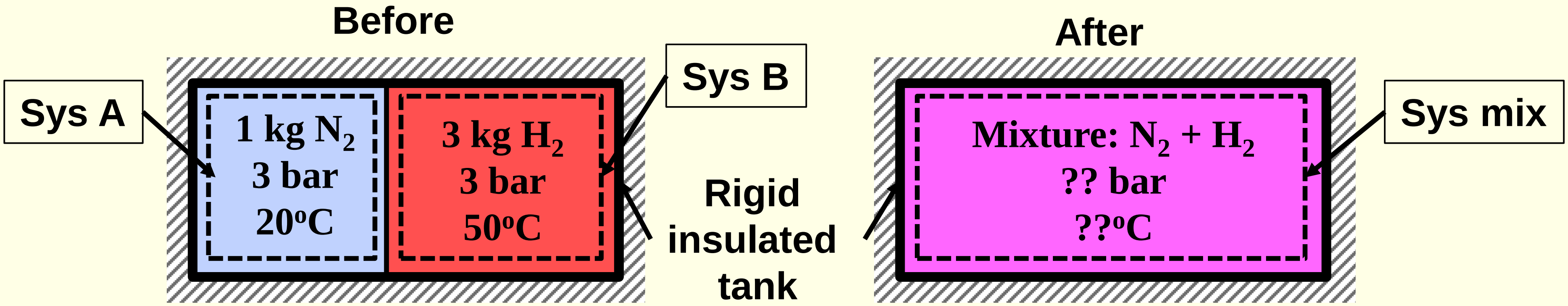
Model(s)?

Ideal gas? Semi? Real?

Extensive properties given?

None? Only one? More than one?

Example 1: Mixing gases



Before

Sys A

1 kg N_2
3 bar
20ºC

Sys B

3 kg H_2
3 bar
50ºC

Rigid insulated tank

After

Sys mix

Mixture: $N_2 + H_2$
?? bar
??ºC

System(s): A, B & Mix (closed)

States: A, B complete; mix ?

Conservation(s): mass & Energy

Process(es): ?? $V_A + V_B = V_{mix}$

Energies: Internal energies

Model(s): Ideal gas $PV = mRT;\ R = \overline{R}/\mu$ $U = mc_vT$

Extensive: m_A, m_B

$$m_A + m_B = m_{mix}$$

$$U_A + U_B = U_{mix}$$

$$x_A = m_A / m_{mix} = \checkmark;\ x_B, R_{mix}, c_{vmix}\ \checkmark$$

$$P_AV_A = m_AR_AT_A \Rightarrow V_A = \checkmark$$
$$P_BV_B = m_BR_BT_B \Rightarrow V_B = \checkmark$$
$$V_{mix} = V_A + V_B = \checkmark$$

$$U_A = m_Ac_{vA}T_A = \checkmark$$
$$U_B = m_Bc_{vB}T_B = \checkmark$$
$$U_{mix} = U_A + U_B = \checkmark$$

$$U_{mix} = m_{mix}c_{vmix}T_{mix} \Rightarrow T_{mix}\ \checkmark$$

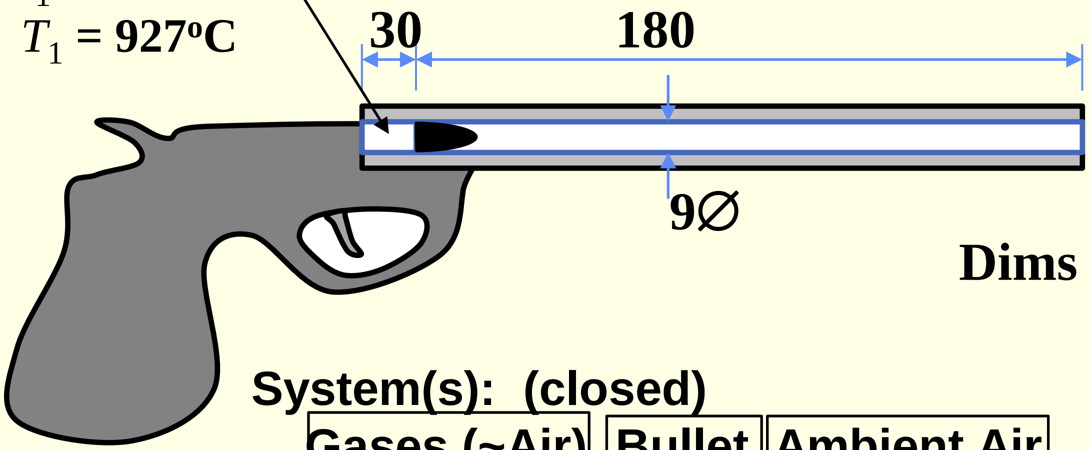
$$P_{mix}V_{mix} = m_{mix}R_{mix}T_{mix} \Rightarrow P_{mix}\ \checkmark$$

Example 2: The gun

Gases ~ Air

P_1 = 100 bar

T_1 = 927°C



Bullet mass = 30 g

v_1= 0; v_2=?? m/s

Ambient Air:

P_{atm} = 1 bar

Dims in mms

System(s): (closed)

Gases (~Air), Bullet, Ambient Air

Conservation(s): Energy

Energies:

- Gas: Internal & Work (W_{gas})
- Bullet: Work (W_{bullet}) & kinetic energy
- Ambient air: W_{amb}

States:

Gases: initial: $P_1, T_1, V_1 \Rightarrow$ complete

Final: V_2 ✓

Process: Gases: adiabatic

$$PV^{\gamma} = Const.$$

Model(s): Ideal gas

Extensive: V_{gas1}, V_{gas2}, m_B

$$P = P_1 (V_1/V)^{\gamma} \Rightarrow P_2 = \checkmark$$

$$Ambient: W_{amb} = P_{atm}(V_2 - V_1) = \checkmark$$

$$W_{gas} = -\int_{V_1}^{V_2} P dV = (P_2V_2 - P_1V_1)/(\gamma - 1) = \checkmark \leftarrow$$

$$Bullet: Q + W = \Delta KE; \quad 0 + W_{bullet} = -W_{gas} - W_{amb} = (m_{bullet}/2)(\mathbf{v}_2^2 - 0); \quad \Rightarrow \mathbf{v}_2 = \checkmark (127 m/s)$$

To retain of chapter 5

Important special cases:

Closed systems

If: only (heat & work of changing volume) & ideal gases:

***Isochoric** ($V=C$, $P \propto T$, $w=0$), **Isobaric** ($P=C$, $V \propto T$, $w=-P\Delta v$), **Isothermal** ($T=C$, $P \propto 1/v$, $w=-RT \ln(v_2/v_1)$*

***Adiabatic** ($q=0$, if no friction: $Pv^{\gamma}=C$), **Polytropic** ($Pv^n=C$, if $n\neq 1$: $w=R(T_2 - T_1)/(n-1)$; …*

Open systems + Steady flow

Throttling: $\boxed{\Delta h=0}$

Flow in ducts: usually $q=w=0 \Rightarrow$ Liquid: $\boxed{\Delta P/\rho + \Delta v^2/2 + \Delta g\, z = -\Delta\, u}$ ***Gases:*** $\boxed{\Delta h + \Delta v^2/2 = 0}$

Heat exchangers: usually $q\neq 0$, $w=0 \Rightarrow$ for each side: $q=\Delta h$

Fluid machines: usually $q=0$, $w\neq 0 \Rightarrow$ $w=\Delta h$ (may by + $\Delta v^2/2$ or – q).

Turbine, pump, compressor, blower, fan

Open systems + Un-Steady flow

$d(.)/dt\neq 0$, $m_{in}\neq m_{out} \Rightarrow$ consider the full First Law

Methodology: the 7 questions:

(System? Conservation? Energies?); (States? Process? Model?); Extensive?

Geothermal Energy Capacity Building in Egypt (GEB)

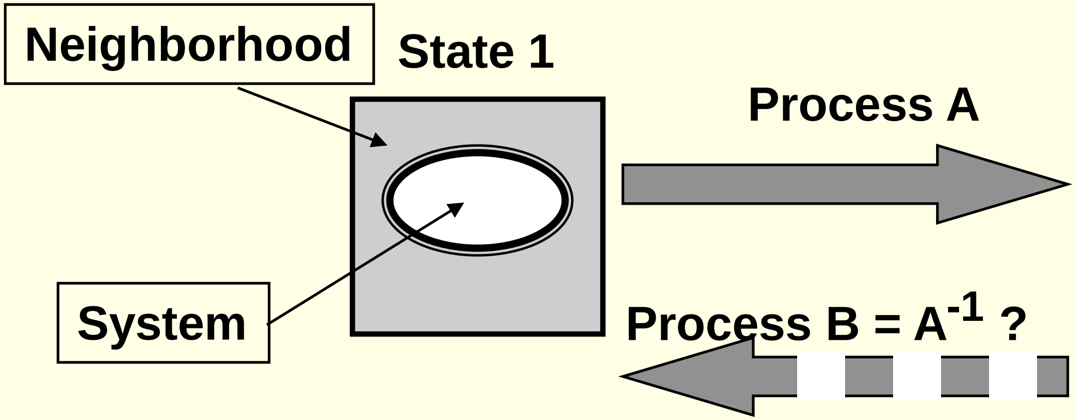
Thermal Engineering for Geothermal Energy

6 – Second Law

Co-funded by the
Erasmus+ Programme
of the European Union

Definition

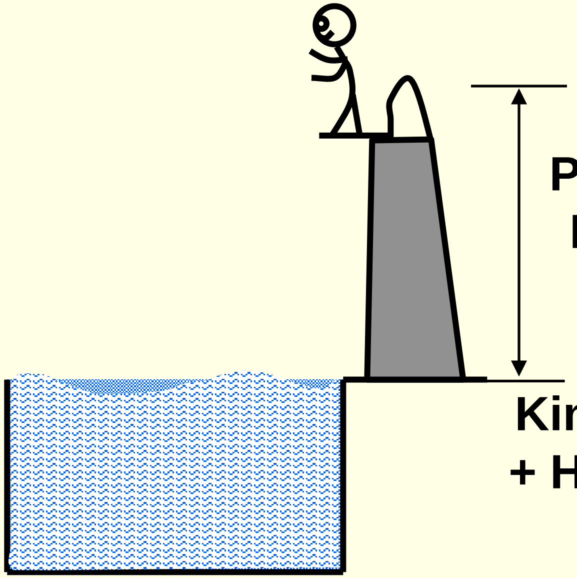
Process A is said to be **Reversible** if there exists another process B allowing **System AND Neighborhood** to restore their **initial states**



Non equilibrium process, Dissipation ⇨ **Irreversible Process**

Sources of irreversibility – 1

A - Friction, viscosity



Potential Energy *PE*

↓

Kinetic Energy *KE* + Heat (air friction)

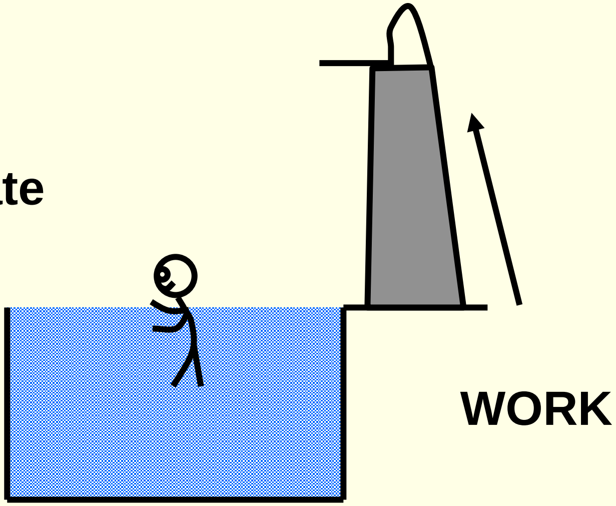
↓

Kinetic Energy *KE* of waves

↓

Heat (viscous friction of water)

To come back To the initial state ⇨

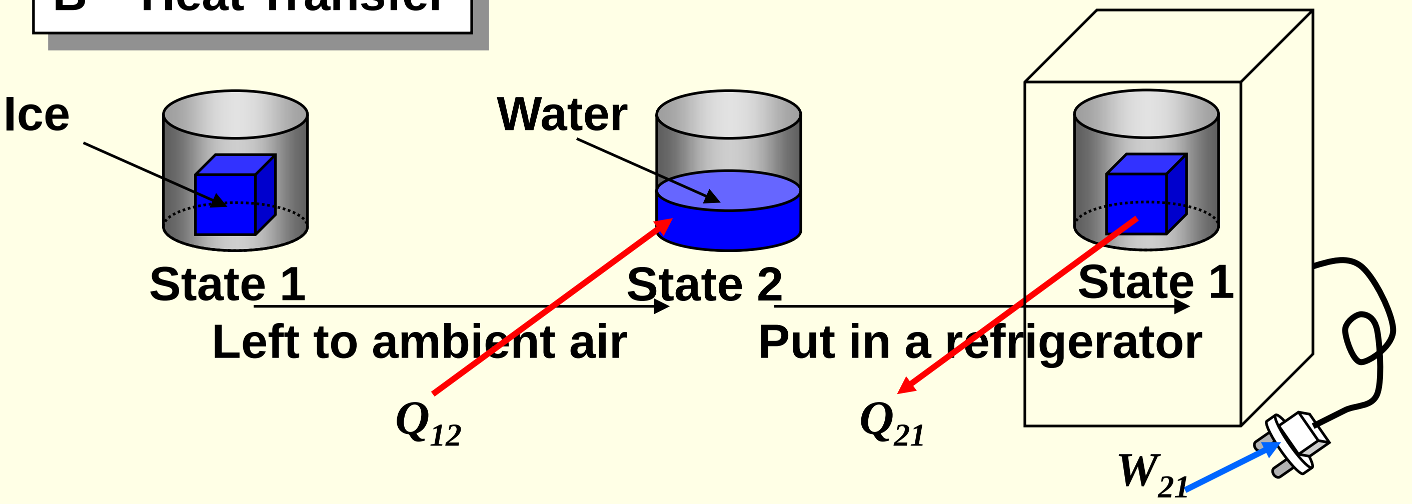


Final Balance :

System: restored initial state
Environment:
- **Gained heat,**
- **Lost work** ⇨ ***Irreversible***

Sources of irreversibility – 2

B – Heat Transfer



<u>Process 1 2 1</u>:

<u>System</u>:

$\Delta U = 0$ ok

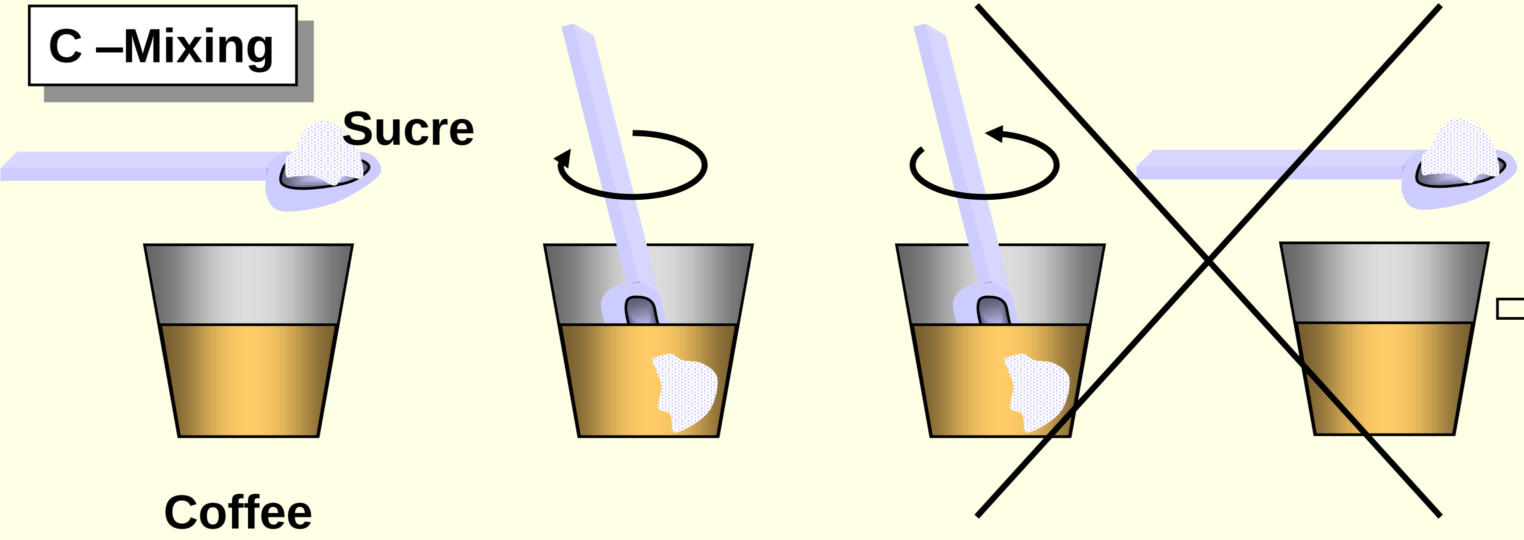
<u>Neighborhood</u>:

$W_{21} > 0$

$Q_{12}+Q_{21} > 0$

⇨ **IRREVERSIBLE**

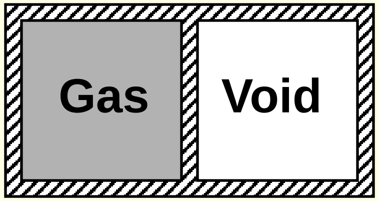
C –Mixing



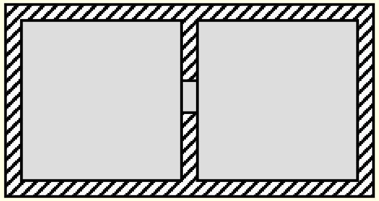
⇨ **IRREVERSIBLE**

Sources of irreversibility – 3

D – Free Expansion



State 1

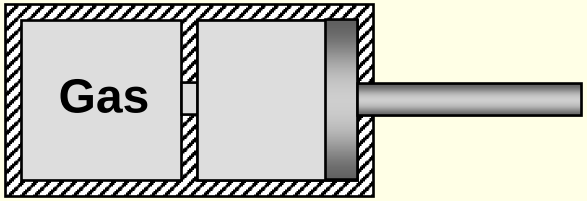


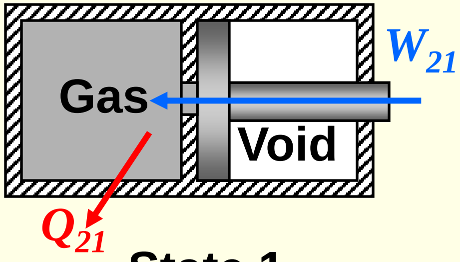
State 2

$U_2 = U_1$; $(Q_{12}=W_{21}=0)$

⇩

To restore Initial state





State 1

***Process 1 2 1*:**

System:

$\Delta U = 0$ **ok**

Neighborhood:

$Q_{21} > 0$

$W_{21} > 0$

⇨ **IRREVERSIBLE**

Definitions

A **Heat reservoir** is a body of high thermal capacity, allowing it to **exchange heat** **without** changing its **temperature**

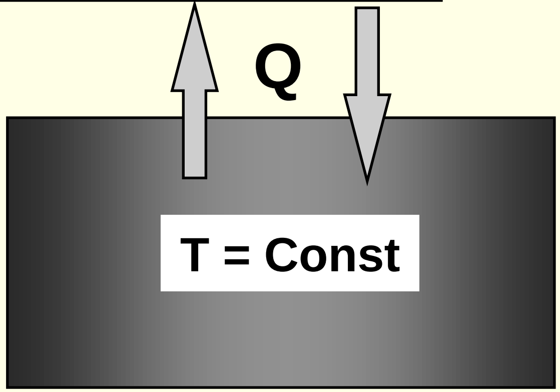
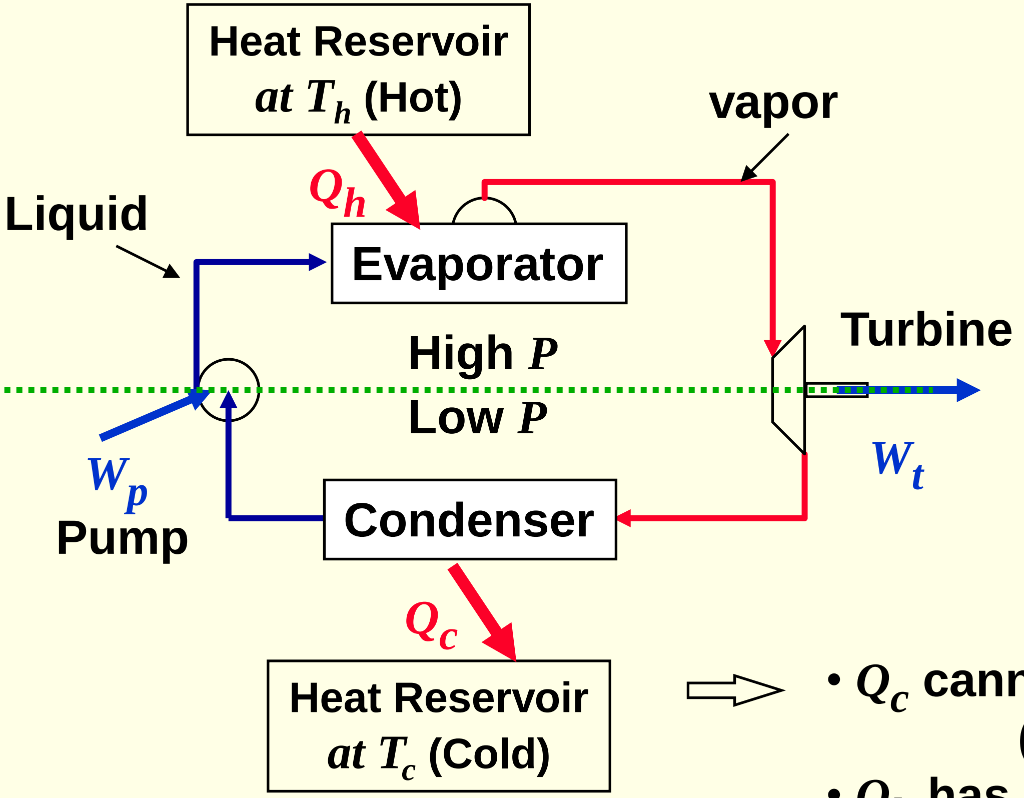


Image:
Water taken from or given to sea
Does not change its level!

A **Heat engine** is a device working on a **Cycle** to exchange **Heat** ⇔ **Work**

1- Motor



Steam Power Plant

$$|Work| = |\int V\, dP|$$

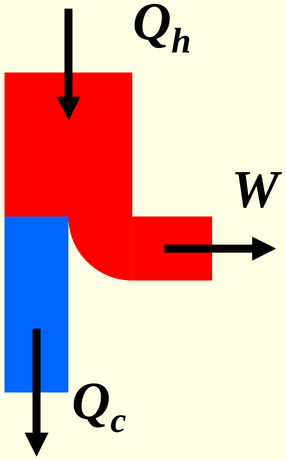
$$(V_{gas} >> V_{liq})$$

$$|W_t| >> |Wp|$$

$$W = |W_t| - |W_p| > 0$$

$$|Q_h| - |Q_c| = W$$

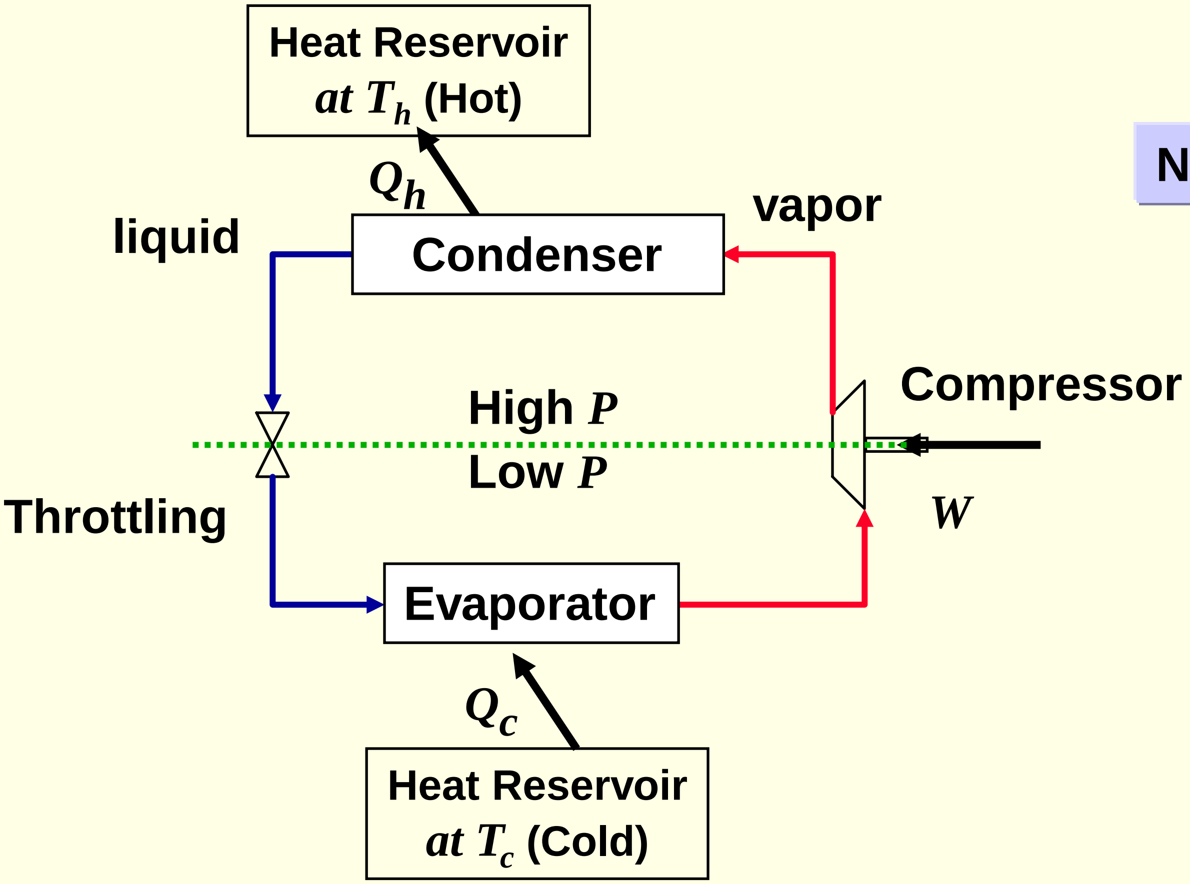
(1st Law)



- Q_c cannot be reused in the cycle
  $(T_c < T_h)$
- Q_h has not been entirely transformed into work

We define Efficiency: $\boxed{\eta = |W| / |Q_h| < 1}$

2- Refrigerator, Heat Pump



Heat Reservoir
at T_h (Hot)

Q_h

liquid

Condenser

vapor

High P

Low P

Compressor

W

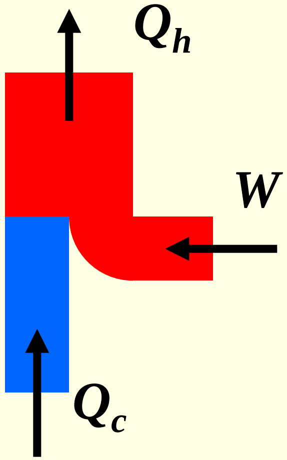
Throttling

Evaporator

Q_c

Heat Reservoir
at T_c (Cold)

NB: $|Q_h| = |W| + |Q_c|$



Coefficient of performance *COP*

Refrigerator

$$COP_{refrig} = |Q_c| / |W|$$

Heat Pump

$$COP_{hp} = |Q_h| / |W| > 1$$

Second Law Statements

A- General

All real processes are ***irreversible***

B - Clausius

Heat ***CANNOT*** be spontaneously transferred
From a cold body to a hotter body

C - Kelvin - Plank

It is ***impossible*** to construct an engine that:

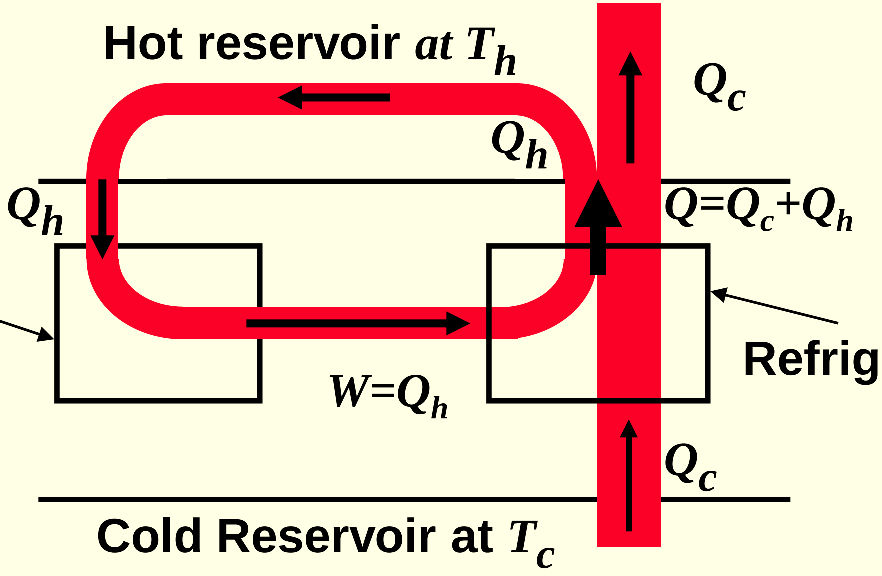
- Works in a cycle
- Produces work
- Exchanges heat with only one reservoir

All together

Equivalence of statements 1

Suppose that we can violate Kelvin-Plank stat. ⇒ **We would violate Clausius stat.!**

Proof:



A hypothetic Motor that violates Kelvin-Plank statement

Refrigerator

<u>If we add a Refrigerator</u>
Receiving $|Q_c|$, $|W| = |Q_h|$,
Giving $|Q| = |Q_c| + |W|$

The heat Q_h can compensate that consumed By the motor

⇓

Heat Q_c has been transferred from T_c to T_h Without external intervention!

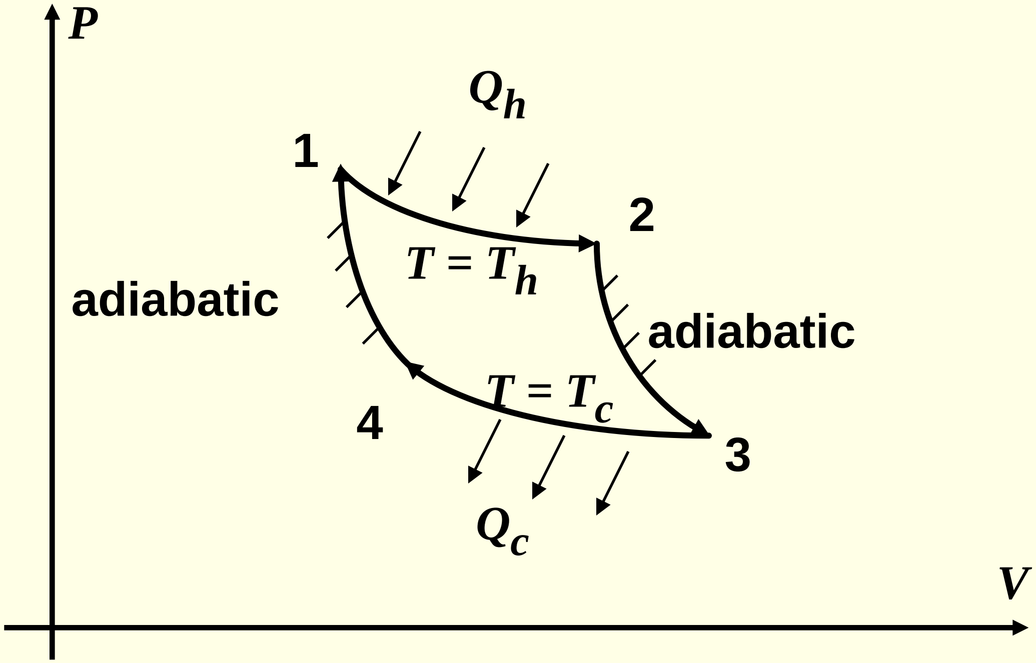
(which violates Clausius statement)

Energy balance:
<u>Hypothetic motor violating K-P</u>:
receives $|Q_h|$, gives $|W| = |Q_h|$

Carnot Cycle

Reversible Cycle

- **Process 1-2 Isothermal**
  - $T = T_h$
  - **|Heat added|** = Q_h
- **Process 2-3 Adiabatic**
  - **Heat = 0**
- **Process 3-4 Isothermal**
  - $T = T_c$
  - **|Heat rejected|** = Q_c
- **Process 4-1 Adiabatic**
  - **Heat = 0**

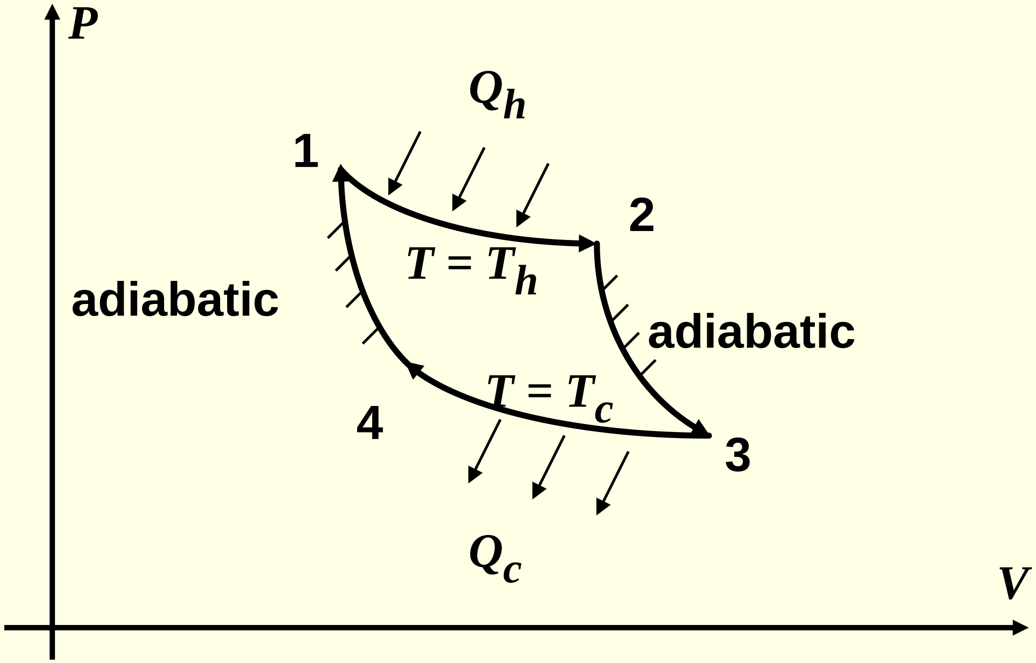


Net Work $|W| = |Q_h| - |Q_c|$

Carnot Cycle for a perfect gas

For a perfect gas:

$PV = m\,R\,T;\;\; dU = mc_v\,dT$



Process 1 - 2:

$|Q_h| = |W_{12}| = mR\,T_h \ln (V_2/V_1)$

Process 3 - 4:

$|Q_c| = |W_{34}| = mR\,T_c \ln (V_3/V_4)$

⇨ $\boxed{\eta = (|Q_h| - |Q_c|)\,/|Q_h| = 1 - |Q_c|\,/|Q_h|}$

⇨ $\boxed{\eta = 1 - (T_c\,/T_h\,)\,[\ln (V_3/V_4)\,/\,\ln (V_2/V_1)]}$

For adiabatic processes:

$$\boxed{T_2\,/\,T_3 = (V_3\,/\,V_2)^{(\gamma-1)} = T_1\,/\,T_4 = (V_4\,/\,V_1)^{(\gamma-1)}}$$

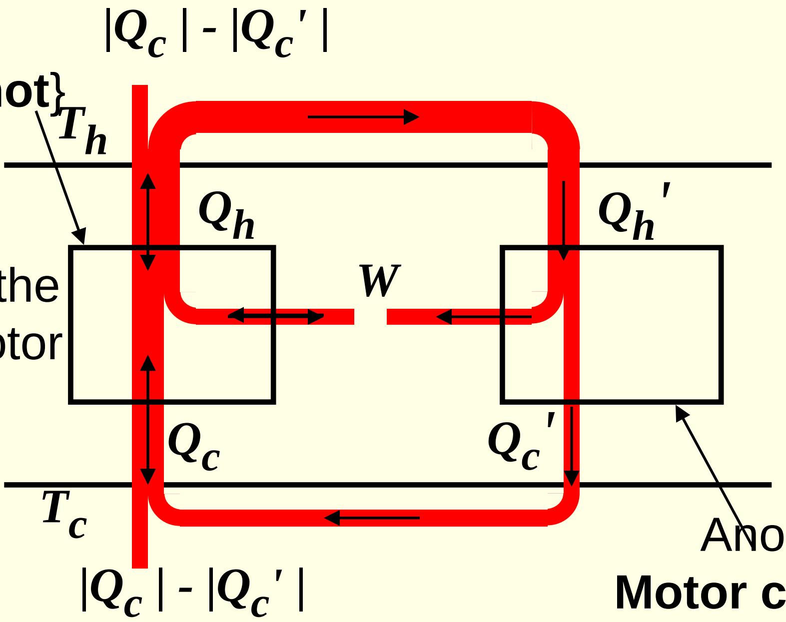
⇨ $V_3\,/\,V_4 = V_2\,/\,V_1$ ➡ $\boxed{\eta = 1 - T_c\,/T_h}$ ➡ $\boxed{\begin{array}{l}|Q_c|\,/\,|Q_h| = T_c\,/\,T_h \\ |Q_c|\,/\,T_c = |Q_h|\,/\,T_h\end{array}}$

Carnot Principles - 1

1 - "Carnot" is the best!

It is **impossible** to construct a motor operting between **2 reservoirs** having an **efficiency that is better** than a **reversible** motor operating between the same reservoirs

A reversible motor {ex: **Carnot**}



Let us invert the reversible motor {ex: **Carnot**}: **Heat Pump**

Another **Motor claiming a better** η than Carnot **!**

Suppose $\eta' > \eta$ **:**

⇒ $|W/Q_h'| > |W/Q_h|$

⇒ $\begin{cases} |Q_h'| < |Q_h| \\ |Q_c'| < |Q_c| \end{cases}$

By joining the 2 machines:

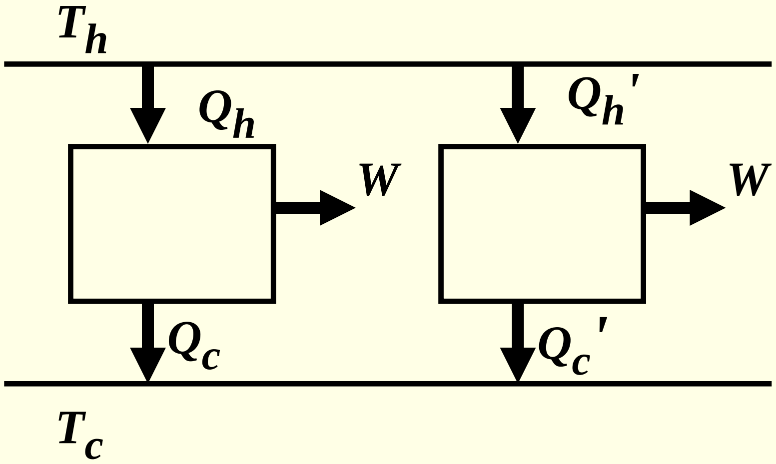
⇒ $|Q_c| - |Q_c'|$ **goes from cold to hot**

<u>Spontaneously</u>**: Impossible!**

Carnot Principles - 2

All other "Carnots" are as good !

All reversible motors working between the **same heat reservoirs** have **the same efficiency**



Proof:

- Put them in parallel,
- Reverse one of them
- Use the preceding principle

⇨ **The efficiency of a reversible motor, working between 2 heat reservoirs depends only on T_h and T_c**
(i.e. not function of the nature matter nor the cycle used)

⇨ $\eta_{reversible} = f(T_c, T_h)\ \forall$ **system matter**
Base for a thermodynamic temperature scale

Performance of heat engines

Notation:

Absolute value of heat exchanged with the hot reservoir: Q_h
Absolute value of heat exchanged with the cold reservoir: Q_c
Absolute value of work: $W = Q_h - Q_c$

For a motor | **For a refrigerator** | **For a heat pump**

By definition, whether the machine was reversible or not:

$\eta = W/Q_h = 1 - Q_c/Q_h \ < 1$

$COP_{ref} = Q_c/W = Q_c/(Q_h - Q_c)$

$COP_{hp} = Q_h/W = Q_h/(Q_h - Q_c) \ \ > 1$

The performance of a reversible machine is always the best possible

$\eta_{reversible} \geq \eta_{irrev}$

$COP_{ref\ reversible} \geq COP_{ref\ irrev}$

$COP_{hp\ reversible} \geq COP_{hp\ irrev}$

For a reversible machine, $Q_h/T_h = Q_c/T_c$, hence:

$\eta_{reversible} = 1 - T_c/T_h < 1$

$COP_{ref\ reversible} = T_c/(T_h - T_c)$

$COP_{hp\ reversible} = T_h/(T_h - T_c) \ \ > 1$

To retain of chapter 6

Reversible= System AND surroundings can restore initial state

Irreversible if: Friction, viscosity, Heat transfer, Mixing, Free expansion

Heat engine: A cycle to exchange Work ⇔ Heat

***Motor**:* Efficiency $\eta = |W| / |Q_h| < 1$

***Refrigerator**:* $COP_{ref} = |Q_c| / |W|$

***Heat pump**:* $COP_{hp} = |Q_h| / |W| > 1$

$|Q_h|$=***Heat exchanged with hot reservoir***

$|Q_c|$=***Heat exchanged with cold reservoir***

$|work|=|Q_h|–|Q_c|$

Second Law Statements (all are equivalent)

***General**:* *All irreversible*

***Clausius**:* *Heat from hot to cold only*

***Kelvin – Plank**:* *No 100% efficiency engine*

$$|Q_h| / T_h = |Q_c| / T_c$$

Carnot cycle: Reversible 4-processes (2 isotherm+2 adiabatic)

- ***Reversible performance ≥ Irreversible performance (all Engines)***
  - ***Reversible Motor:*** $\eta = |Q_h| – |Q_c|) / |Q_h| = 1 – T_c/T_h < 1$
  - ***Reversible Refrigerator:*** $COP= |Q_c| / (|Q_h| – |Q_c|) = T_c / (T_h – T_c)$
  - ***Reversible Heat pump :*** $COP = |Q_h| / (|Q_h| – |Q_c|) = T_h / (T_h – T_c) > 1$

GEB

Geothermal Energy Capacity Building in Egypt (GEB)

Thermal Engineering for Geothermal Energy

7 – Entropy

Co-funded by the
Erasmus+ Programme
of the European Union

Introduction from physics: Clausius Inequality

For a reversible Carnot cycle:

$$\eta_{Carnot} = 1 - T_c / T_h = 1 - |Q_c / Q_h|$$

$$|Q_h / T_h| = |Q_c / T_c|$$

Respecting signes: $\left.(Q_h / T_h + Q_c / T_c)\right|_{rev} = 0$

For an irreversible cycle:

For the same $|Q_h|$: $|Q_c|$ will be greater

$$\left.(Q_h / T_h + Q_c / T_c)\right|_{irrev} < 0$$

An equivalent process

One can replace:

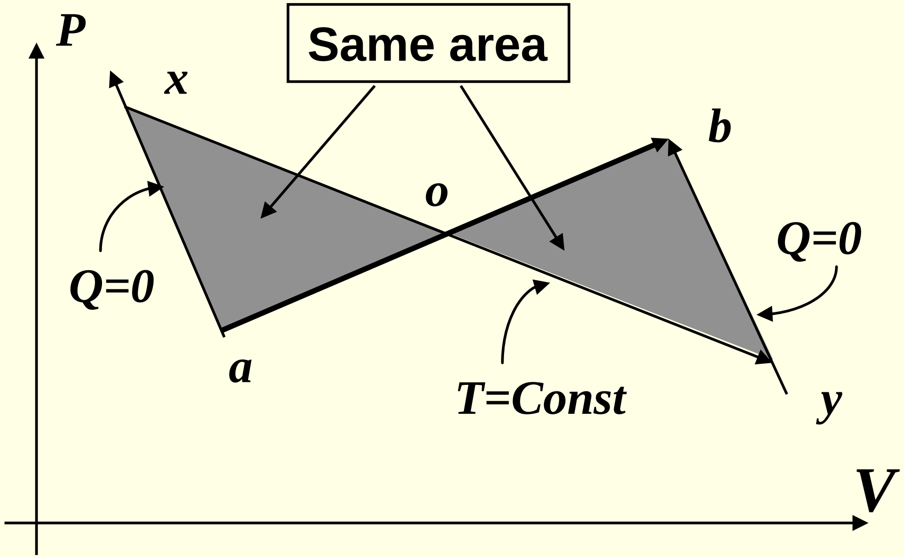
A general process: *ab*

By an "equivalent"

Process *ax – xy – yb*

***ax*: Adiabatic** **yb*: Adiabatic**

***xy*: Isothermal such as same area**



Since area of *axo* = area of *oyb*:

$$W_{ab} = W_{axyb}$$

But: $$\Delta U_{ab} = \Delta U_{axyb}$$

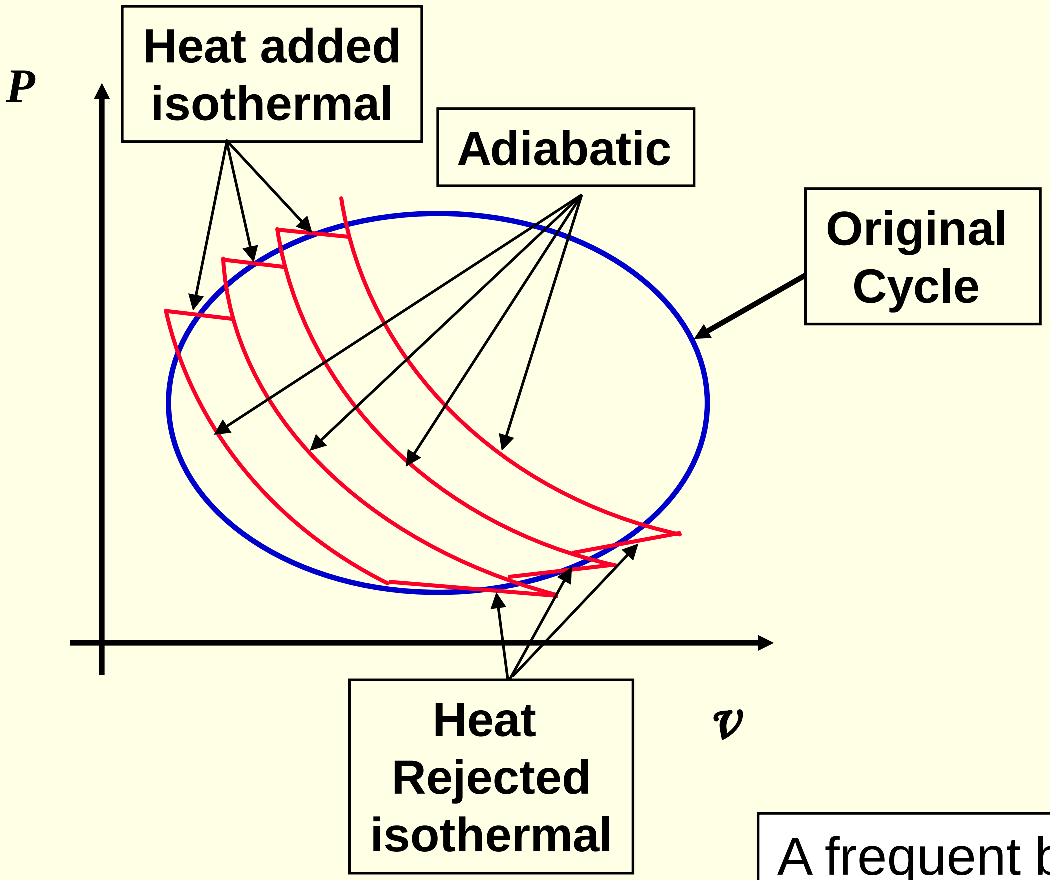
$$\Rightarrow \quad Q_{ab} = Q_{axyb}$$

$\Rightarrow$ **Process *ab* is equivalent to process *axyb***

Clausius Inequality for any cycle

Transform into:

Original Cycle ⇨ **n cycles of Carnot**



Clausius inequality
For the n cycles of Carnot:

$$\Sigma\, Q_i / T_i \leq 0$$

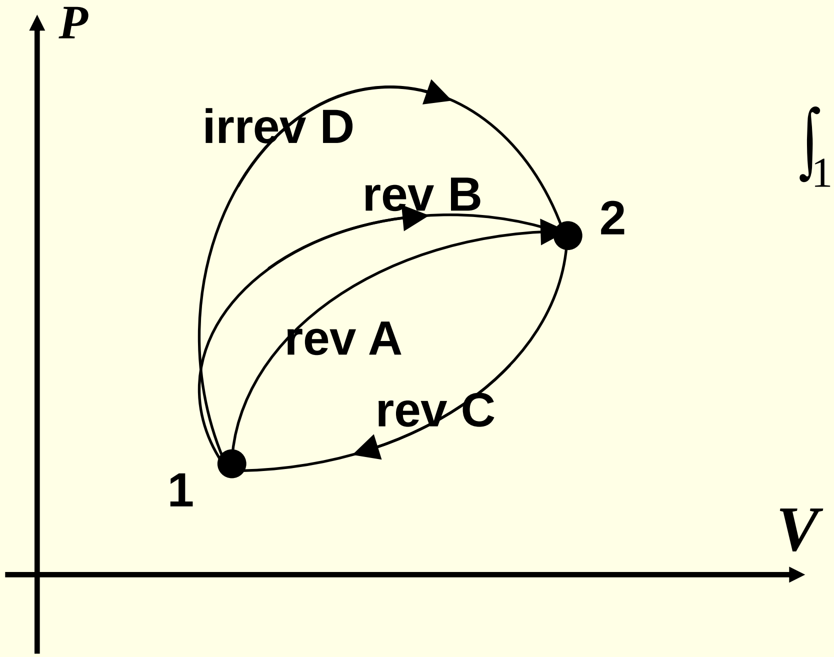
∴ For the original cycle

$$\boxed{\int_{cycle} (\dot{Q} / T)\, dt \leq 0}$$

Equality applies
for reversible cases
ONLY

A frequent but ***illegal*** formulation **:** $\int đQ / T \leq 0$

Entropy starting from Clausius



Equation (1) – (2):

$\int_{12-rev} \dot{Q}/T\,dt$ **Does *NOT* depend on the process As long as it is *reversible***

Define: $$\Delta S_{12} = S_2 - S_1 = \int_{12-rev} \dot{Q}/T\,dt$$

Equation (1) – (3):

$$\int_{12-rev} \dot{Q}/T\,dt - \int_{12-irrev} \dot{Q}/T\,dt \geq 0$$

Applying Clausius Inequality:

Cycle 1A2C1: $\int_{12-revA} \dot{Q}/T\,dt + \int_{21-revC} \dot{Q}/T\,dt = 0$ **(1)**

Cycle 1B2C1: $\int_{12-revB} \dot{Q}/T\,dt + \int_{21-revC} \dot{Q}/T\,dt = 0$ **(2)**

Cycle 1D2C1: $\int_{12-irrevD} \dot{Q}/T\,dt + \int_{21-revC} \dot{Q}/T\,dt \leq 0$ **(3)**

In General:

$$\boxed{\Delta S_{12} = \int_{12} \dot{Q}/T\,dt + \Delta S_{irrev}}$$

$$\boxed{\Delta S_{irrev} \geq 0}$$

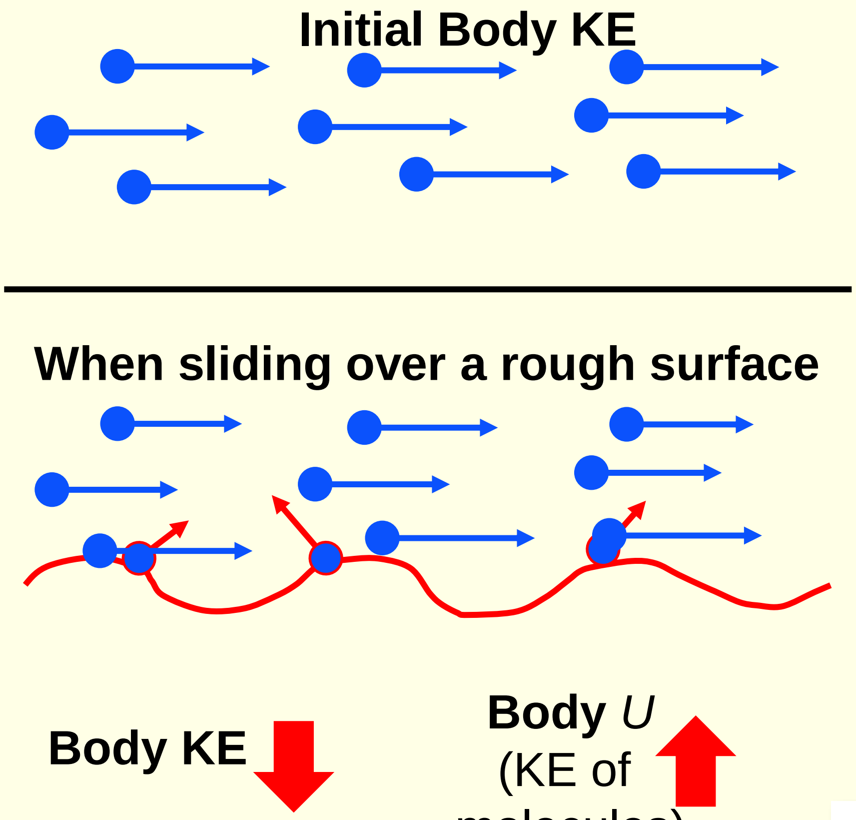
How entropy changes?

$$\Delta S_{12} = \int_{12} \dot{Q}/T \, dt + \Delta S_{irrev}$$

$$\Delta S_{irrev} \geq 0$$

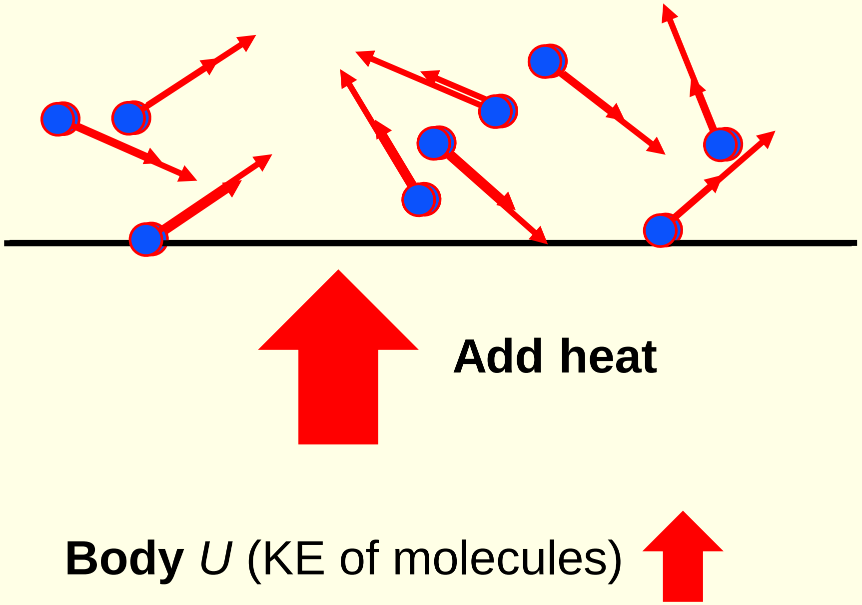
ΔS > 0: Adding heat or irreversible process

ΔS < 0: Rejecting heat



Body KE

Body *U* (KE of molecules)

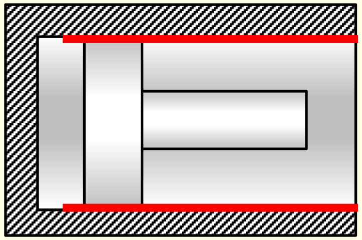


Body *U* (KE of molecules)

First **Partial** understanding:

Increasing **Chaos** increases **Entropy**

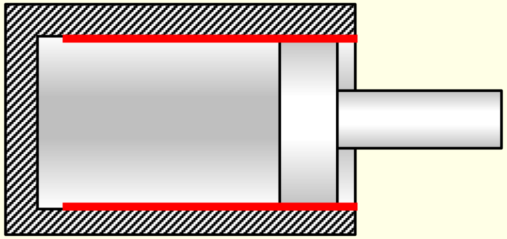
Departure from reversible



Reversible expansion work: $|W_{rev12}|$

IRReversible expansion work: $|W_{12}|$

$$|W_{12}| \geq |W_{rev12}|$$



Reversible compression work: W_{rev12}

IRReversible compression work: W_{12}

$$|W_{12}| \geq |W_{rev12}|$$

$$\boxed{W_{12} \geq W_{rev12}}$$

$$\boxed{\text{“Friction” work: } W_{12} - W_{rev12}}$$

Reversible process: $Q_{rev12} + W_{rev12} = U_2 - U_1$

IRReversible process: $Q_{12} + W_{12} = U_2 - U_1$

Subtracting: $Q_{rev12} - Q_{12} + W_{rev12} - W_{12} = 0$

$$\boxed{S_2 - S_1 = \int_{12} \left(\dot{Q}/T\right) dt + \Delta S_{irrev}} \quad \Longrightarrow \quad \int_{rev12} \left(\dot{Q}/T\right) dt - \int_{12} \left(\dot{Q}/T\right) dt = \Delta S_{irrev}$$

where $\Delta S_{irrev} \geq 0$

Gibbs equation

Consider a 'short' process during time *dt*

Heat **+** **Work** **Internal E**

If reversible:

If irreversible:

$$\left.\begin{array}{ll} \dot{Q}_{rev}\,dt = TdS & \dot{W}_{rev}\,dt = -PdV \\ \dot{Q}\,dt = TdS - TdS_{irrev} & \dot{W}\,dt = -PdV + TdS_{irrev} \end{array}\right\} \quad dU$$

2nd Law **1st Law**

Gibbs Equation:

$$\boxed{T\,dS = dU + P\,dV} \qquad \boxed{T\,dS = dH - V\,dP}$$

Equivalent to:

$$\boxed{S_2 - S_1 = \int_{12} \left(\dot{Q}/T\right)dt + \Delta S_{irrev}}$$

Entropy of different phases

$$s_2 - s_1 = \int_1^2 \frac{du + P dv}{T} = \int_1^2 \frac{dh - v dP}{T}$$

Ideal Gas:

$$s_2 - s_1 = \int_1^2 \left(\frac{c_v dT}{T} + \frac{R dv}{v} \right) = \int_1^2 \left(\frac{c_p dT}{T} - \frac{R dP}{P} \right)$$

$$\gamma/(\gamma - 1)$$

$$s_2 - s_1 = c_v \ln(T_2/T_1) + R \ln(v_2/v_1) = c_P \ln(T_2/T_1) - R \ln(P_2/P_1)$$

$$1/(\gamma - 1)$$

If isentropic: $s_2 - s_1 = 0 \Rightarrow P_2/P_1 = (T_2/T_1)^{\boxed{c_P/R}}; \quad V_2/V_1 = (T_1/T_2)^{\boxed{c_v/R}}$

Semi-Ideal Gas

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_P dT}{T} - R \ln\left(\frac{P_2}{P_1}\right) = s^0(T_2) - s^0(T_1) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$s^0(T) = \int \frac{c_P dT}{T}$$

If isentropic: $s_2 - s_1 = 0 \Rightarrow \dfrac{P_2}{P_1} = \dfrac{P_r(T_2)}{P_r(T_1)}; \quad \dfrac{V_2}{V_1} = \dfrac{V_r(T_2)}{V_r(T_1)}$

where: $P_r(T) = \exp\left(s^0(T)/R\right); \quad V_r(T) = T \exp\left(-s^0(T)/R\right)$

Other cases

Wet Vapor:

$$s = (1 - x) s_f + x\, s_g$$

Solids, liquids: $c_p \cong c_v \cong$ **Constant** c ; $v \cong$ **constant**

$$s_2 - s_1 = c \ln\left(T_2 / T_1\right)$$

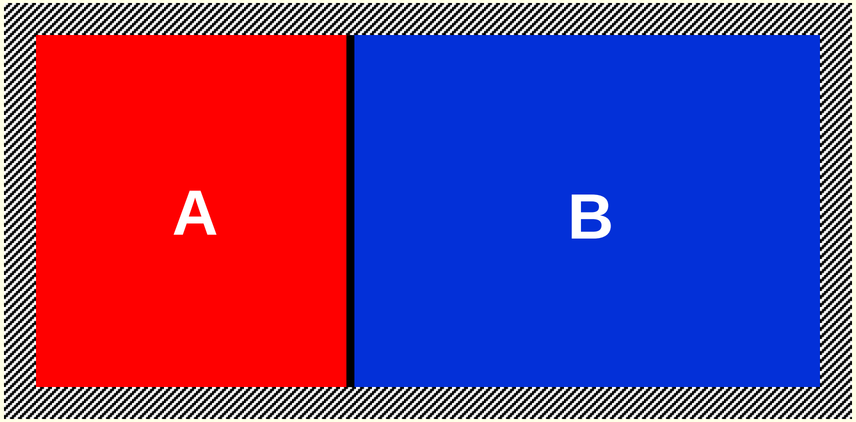
Gas mixtures

$$\Delta S = \Delta S_A + \Delta S_B$$

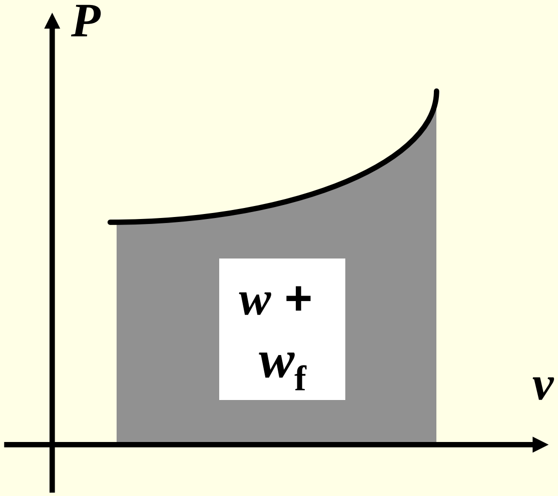
$$\Delta S_A = m_A \left[c_P \ln \frac{T_{after}}{T_{before}} - R \ln \frac{P_{after}}{P_{before}} \right]_A$$

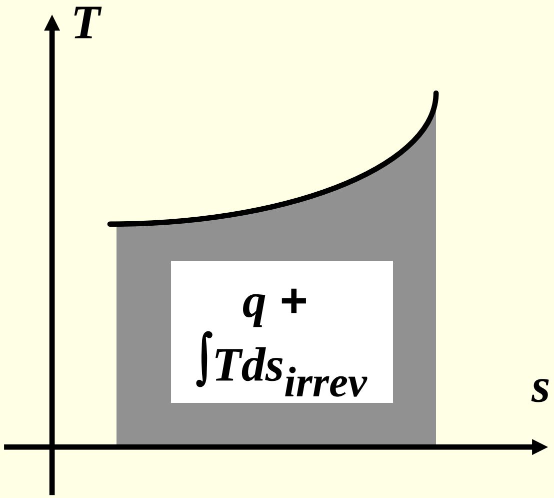
$$\Delta S_B = m_B \left[c_P \ln \frac{T_{after}}{T_{before}} - R \ln \frac{P_{after}}{P_{before}} \right]_B$$

Partial pressure



T- s, h - s Charts





$$T\,ds = \dot{q}\,dt + T\,ds_{irrev}$$

For an ideal gas:

$$\begin{aligned} s_2 - s_1 &= c_v \ln(T_2/T_1) + R\ln(v_2/v_1) \\ &= c_P \ln(T_2/T_1) - R\ln(P_2/P_1) \end{aligned}$$

For *v* = constant:

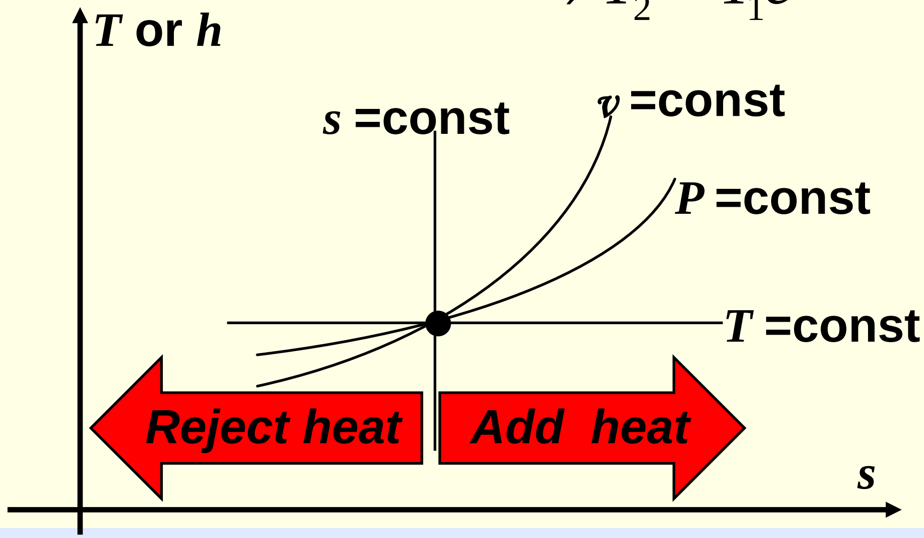
$$s_2 - s_1 = c_v \ln(T_2/T_1)$$

$$\Rightarrow T_2 = T_1 e^{(s_2 - s_1)/c_v}$$

For *P* = constant:

$$s_2 - s_1 = c_P \ln(T_2/T_1)$$

$$\Rightarrow T_2 = T_1 e^{(s_2 - s_1)/c_P}$$



T - s, h - s charts (next)

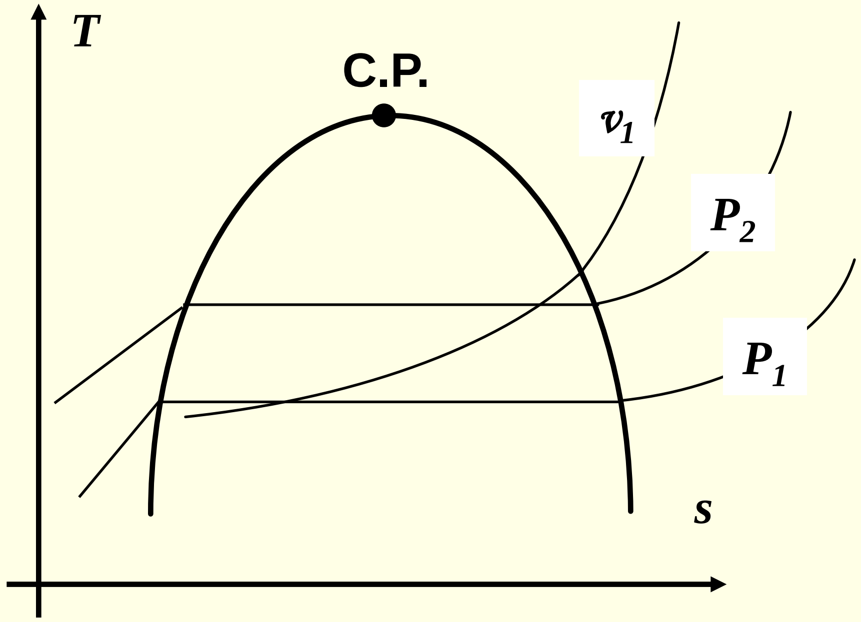
Region of wet vapor

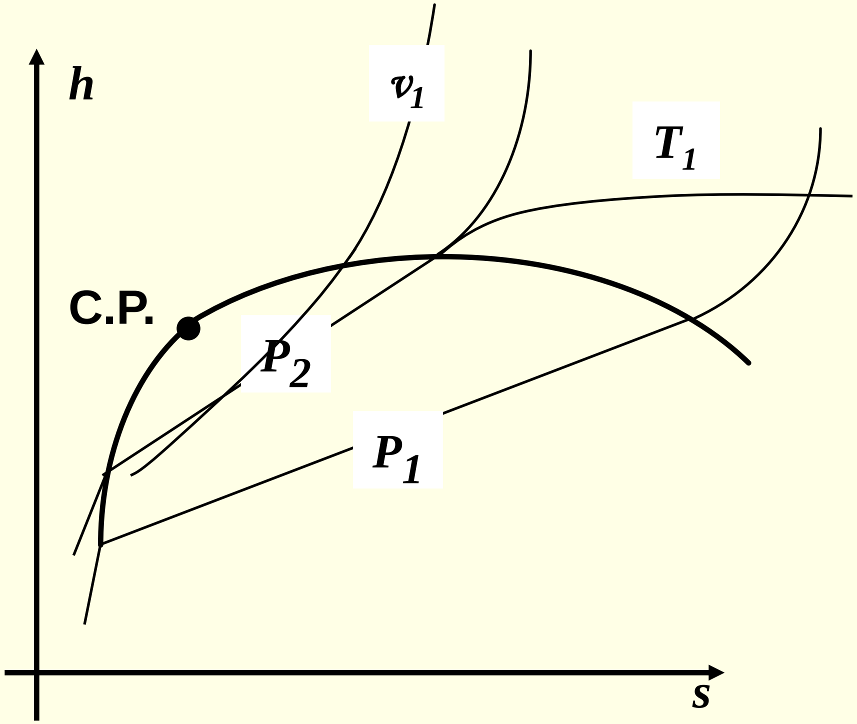
NB:

$P = cont \Rightarrow T = const$

$T\,ds = dh - v\,dP$

$\Rightarrow \partial h / \partial s |_{P\,\mathrm{const}} = T$

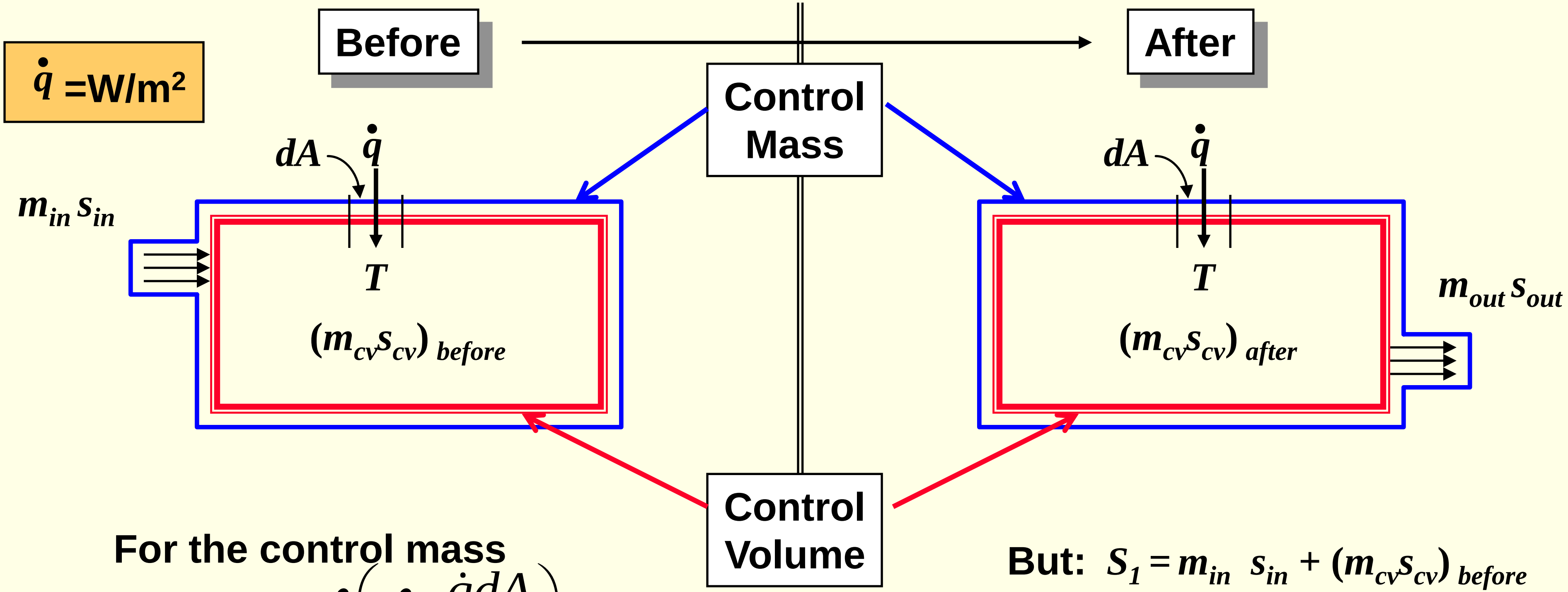




Entropy of a control volume

Additional

$\dot{q}$ =W/m²



For the control mass

$$\Delta S = \underbrace{S_2 - S_1}_{S_{after}-S_{before}} = \int \left(\oint_A \frac{\dot{q}dA}{T} \right) dt + \Delta S_{irrev}$$

But: $S_1 = m_{in}\; s_{in} + (m_{cv}s_{cv})_{before}$

$S_2 = m_{out}\, s_{out} + (m_{cv}s_{cv})_{after}$

⇨ **For the control volume**

$$\Delta\left(m_{cv}s_{cv}\right) + m_{out}s_{out} - m_{in}s_{in} = \int \left(\oint_A \frac{\dot{q}dA}{T} \right) dt + \Delta S_{irrev}$$

$$d\left(m_{cv}s_{cv}\right)/dt + \dot{m}_{out}s_{out} - \dot{m}_{in}s_{in} = \oint_A \frac{\dot{q}dA}{T} + dS_{irrev}/dt$$

Additional

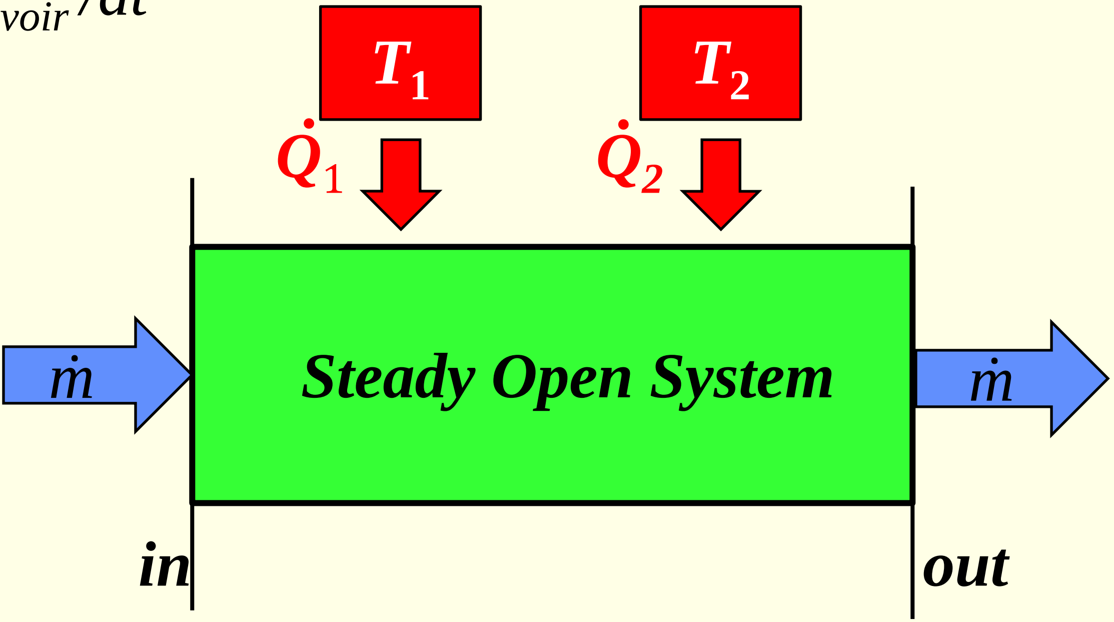
Open steady state steady flow system

$$dS_{universe}/dt = dS_{system}/dt + dS_{fluid}/dt + dS_{reservoir}/dt$$

$$dS_{system}/dt = 0 \text{ (steady)}$$

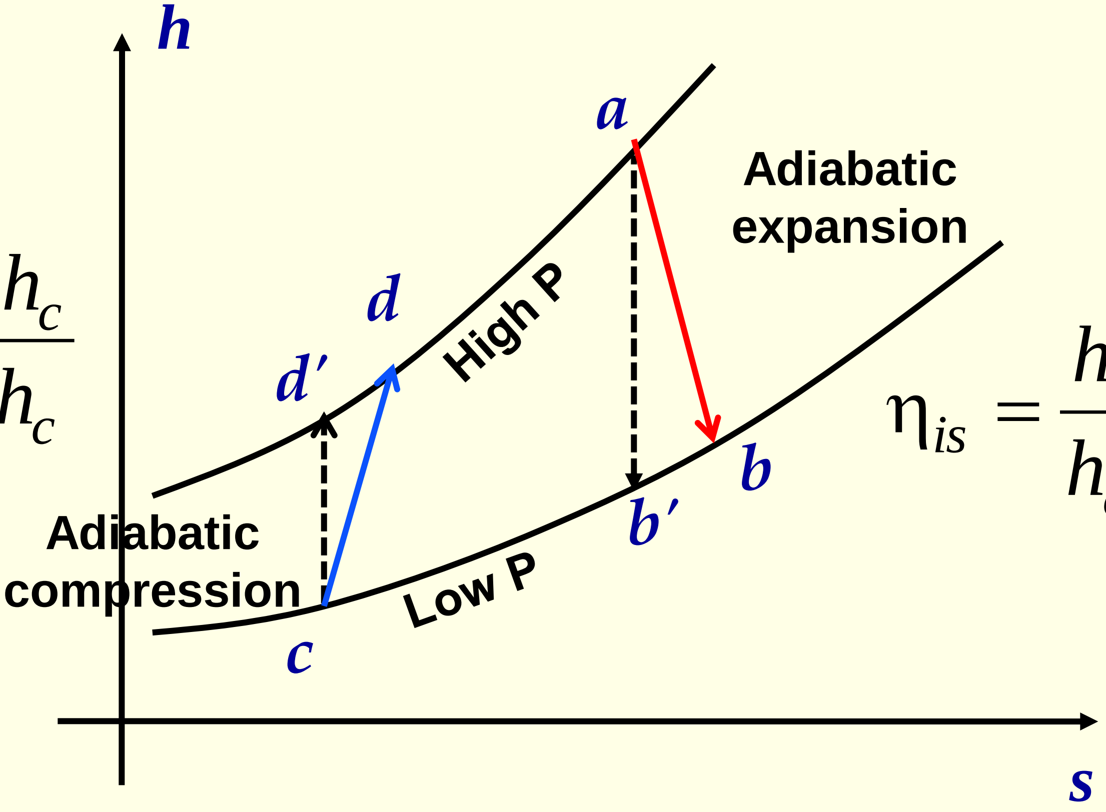
$$dS_{fluid}/dt = \dot{m}\left(s_{out} - s_{in}\right)$$

$$dS_{reservoir}/dt = \sum_i \dot{Q}_i / T_i$$



Isentropic efficiency

$$\eta_{is} = \frac{h_{d'} - h_c}{h_d - h_c}$$



$$\eta_{is} = \frac{h_a - h_b}{h_a - h_{b'}}$$

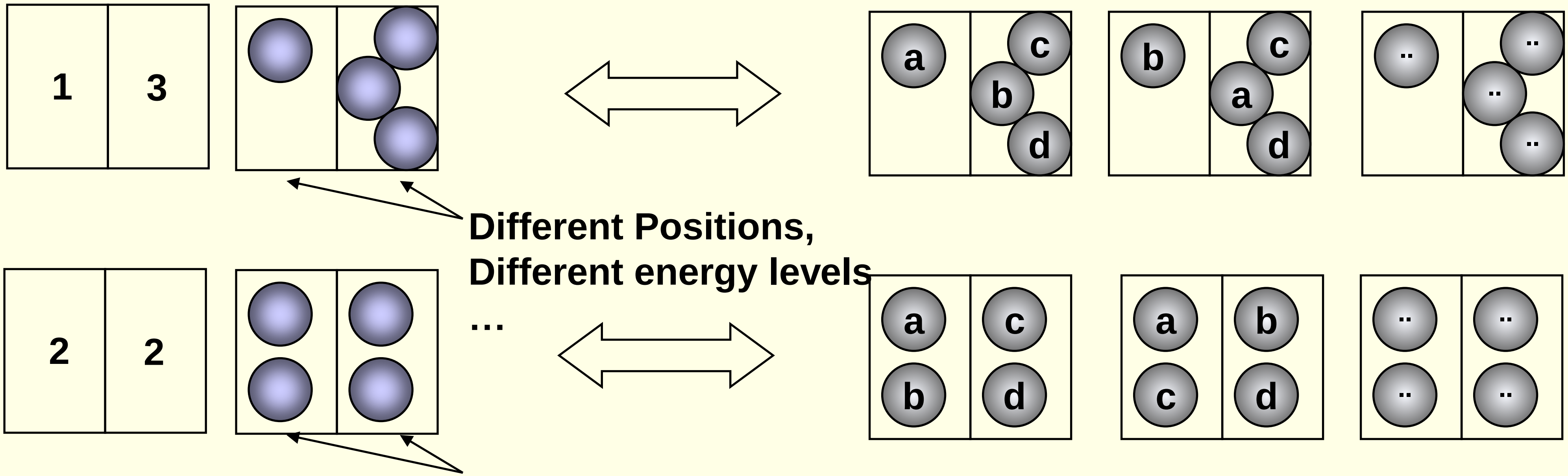
Statistical introduction: Micro & Macro states

Macro - state

Distribution assuming
indistinguishable objects

Micro - states

Distribution assuming
distinguishable objects



Different Positions,
Different energy levels
…

Major Assumptions:

- **The distribution of molecules is <u>totally</u> random**
- **All micro-states are *<u>equally</u>* probable**

Thermodynamic probability W

The thermodynamic probability W **of a macro state**
= Number of corresponding *micro states*

A given Macro state

Box 1 N_1 **Box 2** N_2 … **Box** J N_J

$$N = \sum_{j=1}^{J} N_j$$

To get corresponding Micro state:

Number of arrangements of N distinguishable molecules : $N!$

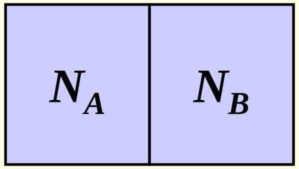
Number of arrangements of N_j distinguishable molecules in box j: $N_j!$

$$\boxed{W = \frac{N!}{N_1! N_2! \ldots N_J!}}$$

Distribution between 2 boxes

Macro state

N_A - N_B	Micro states												W
0 - 4		1,2,3,4											1
1 - 3	1	2,3,4	2	1,3,4	3	1,2,4	4	1,2,3					4
2 - 2	1,2	3,4	1,3	2,4	1,4	2,3	3,4	1,2	2,4	1,3	2,3	1,4	6
3 - 1	2,3,4	1	1,3,4	2	1,2,4	3	1,2,3	4					4
4 - 0	1,2,3,4												1

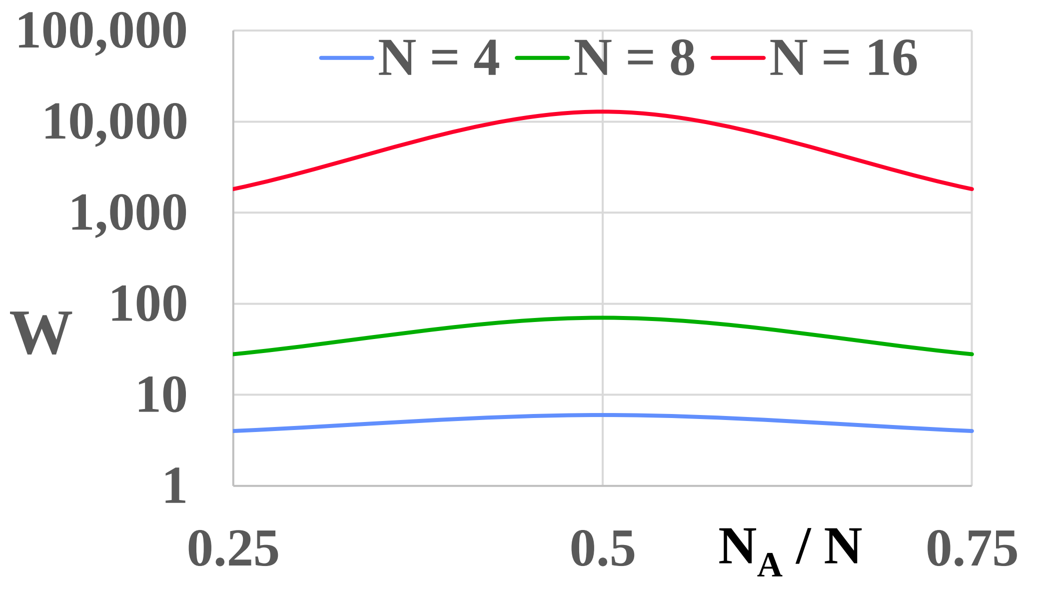


$$W = \frac{N!}{N_A! N_B!}$$

For large *N* use Sterling formula:

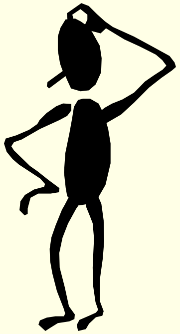
$$\boxed{\ln N! \approx N \ln N - N}$$

⇨ $$W \approx (N/N_A)^{N_A} . (N/N_B)^{N_B}$$



Entropy starting from statistics

- **The distribution of molecules is totally random**
- **All micro-states are *equally* probable**
- **The macro-state of W max is the *most probable***
- ***An isolated system tends spontaneously towards the state of W max***



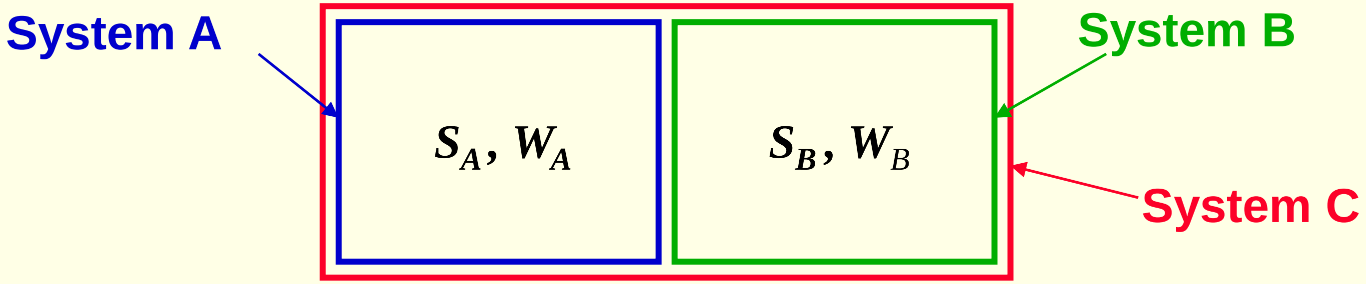
Another formulation of Second Law (direction of a process):

An isolated system spontaneously tends towards W max

$$S = f\,(W)$$

Relation between entropy S and probability W

$$S \overset{?}{=} f(W)$$



$$S_C = S_A + S_B$$

$$S_C = S_A + S_B \qquad W_C = W_A \,.\, W_B \quad \Rightarrow \quad f(W_A W_B) = f(W_A) + f(W_B)$$

$\partial / \partial W_A$

$$W_B f'(W_A W_B) = f'(W_A)$$

$\partial / \partial W_B$

$$W_A W_B f''(W_A W_B) + f'(W_A W_B) = 0$$

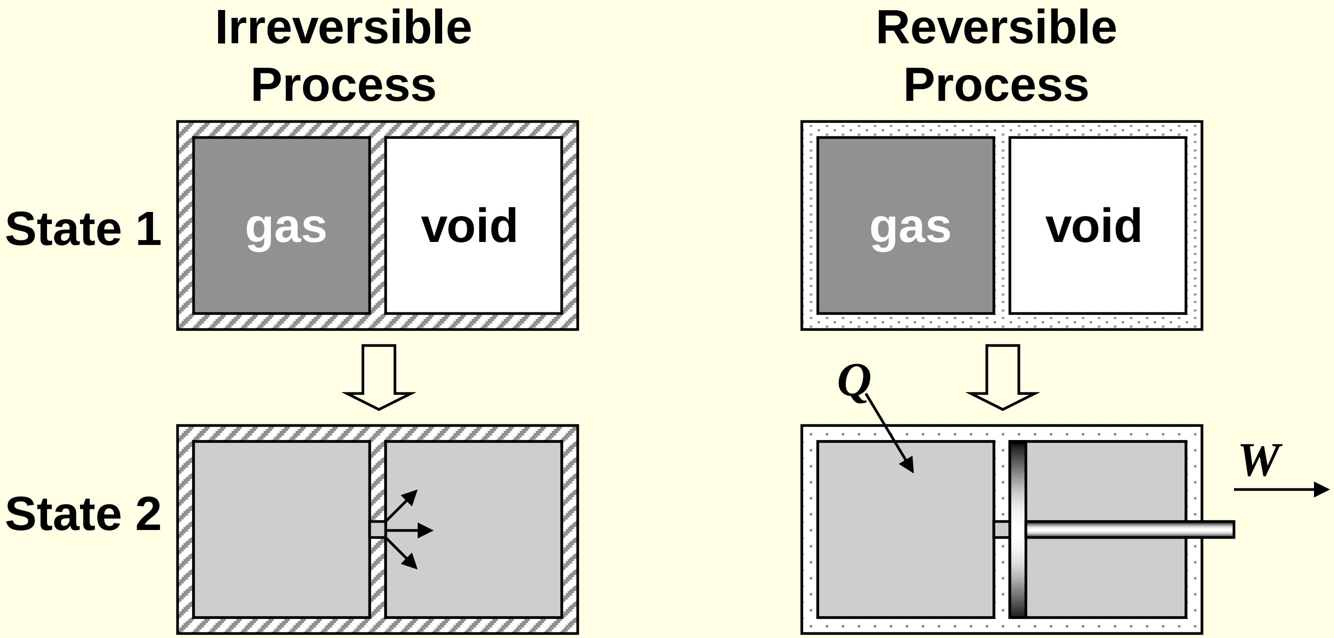
$$W f''(W) + f'(W) = 0$$

$$S = f(W) = C_1 \ln(W) + C_2$$

Put: $S = 0$ for $W=1$

$$S = C_1 \ln(W)$$

Obtaining the coefficient



Quantity of gas = 2 mol
= 2 N_A molecules

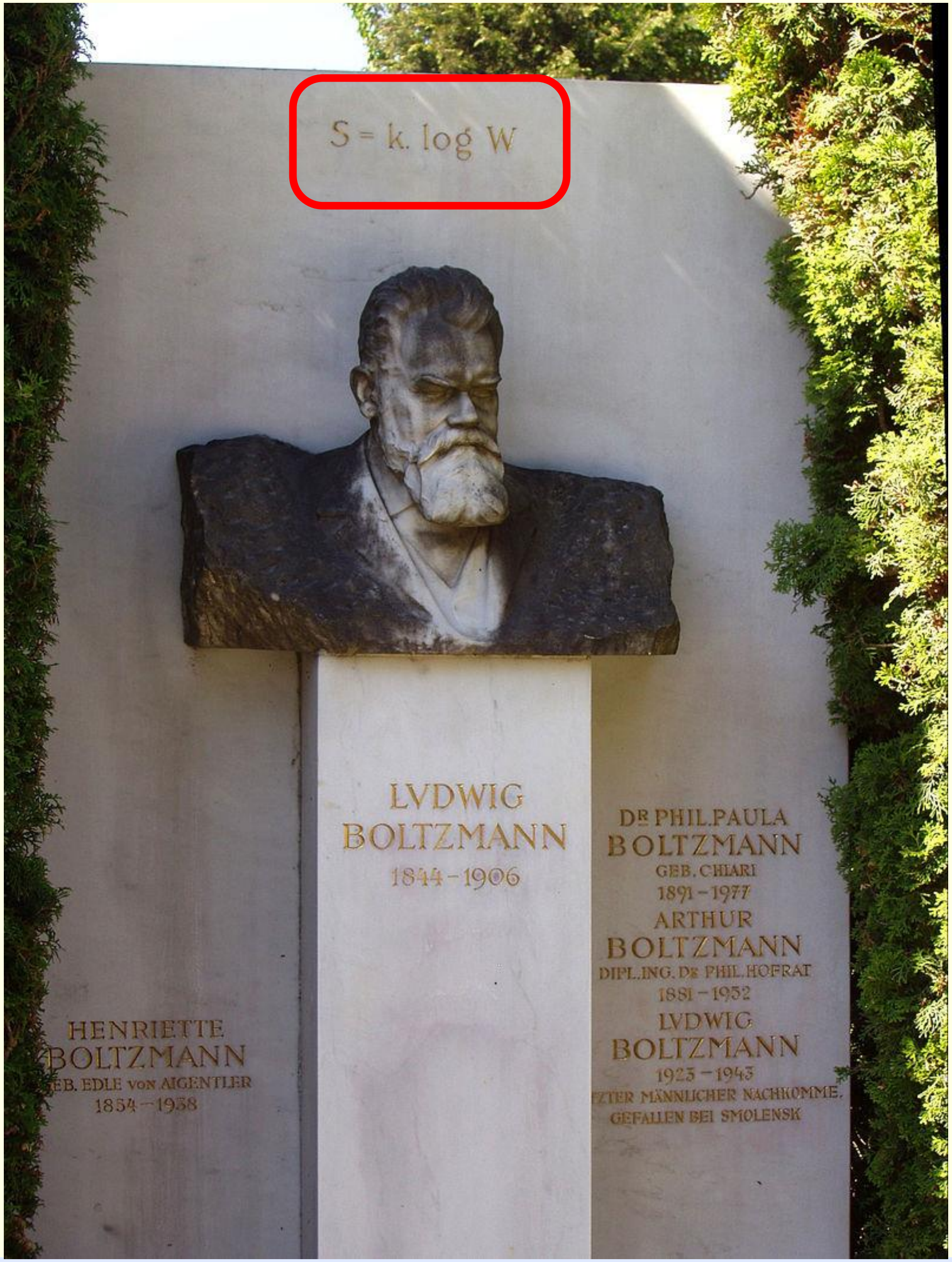
$Q = W$

$$\boxed{\Delta S_{12} = \Delta S_{irrev}} \Longleftrightarrow \boxed{\Delta S_{12} = \int_{12\_rev} \frac{\dot{Q}}{T}\,dt = \frac{Q_{12\_rev}}{T} = \frac{2\bar{R}T\ln\left(V_2/V_1\right)}{T} = 2\bar{R}\ln(2)}$$

$$\left.\begin{array}{l} W_1 = 1 \\ W_2 = \dfrac{(2N_A)!}{N_A!\,N_A!} \end{array}\right\} \boxed{\Delta S_{12} = C_1\left[\ln\left(W_2\right) - \ln\left(W_1\right)\right] \approx C_1\left[2N_A \ln 2\right]}$$

$C_1 = \bar{R}/N_A = k$

$S = k \ln W$


S = k. log W
LVDWIG
BOLTZMANN
1844 – 1906
DR PHIL. PAULA
BOLTZMANN
GEB. CHIARI
1891 – 1977
ARTHUR
BOLTZMANN
DIPL. ING. DR PHIL. HOFRAT
1881 – 1952
LVDWIG
BOLTZMANN
1923 – 1943
TZTER MÄNNLICHER NACHKOMME,
GEFALLEN BEI SMOLENSK
HENRIETTE
BOLTZMANN
EB. EDLE von AIGENTLER
1854 – 1938

To retain of chapter 7

Clausius inequality for any general cycle:

$$\oint_{cycle} \frac{\dot{Q}}{T}\,dt \le 0$$

$$\cancel{\oint_{cycle} \frac{đQ}{T} \le 0;\ \oint_{cycle} \frac{\delta Q}{T} \le 0}$$

Entropy for a general process:

$$\Delta S_{12} = \int_{12} \dot{Q}/T\,dt + \Delta S_{irrev}$$

$$\Delta S_{irrev} \ge 0$$

ΔS > 0: Adding heat or irreversible process

ΔS < 0: Rejecting heat

Gibbs Equation

$$T\,dS = dU + P\,dV$$

$$T\,dS = dH - V\,dP$$

ΔS for ideal gas:

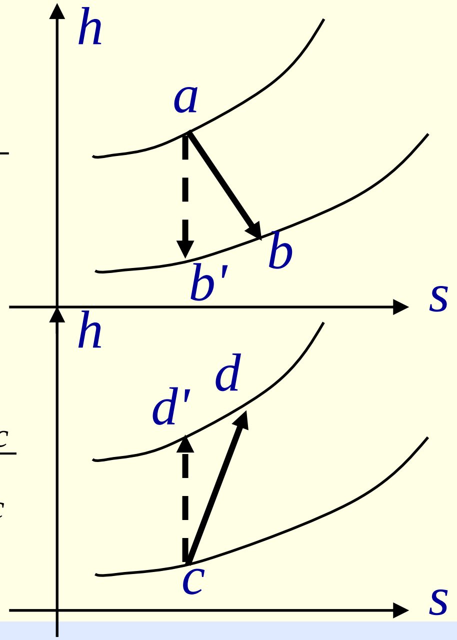
$$s_2 - s_1 = c_v \ln\left(T_2/T_1\right) + R\ln\left(v_2/v_1\right) = c_P \ln\left(T_2/T_1\right) - R\ln\left(P_2/P_1\right)$$

For an OPEN system: Added terms:

$$d\left(m_{cv}s_{cv}\right)/dt + \dot{m}_{out}s_{out} - \dot{m}_{in}s_{in} = \oint_A \frac{\dot{q}\,dA}{T} + dS_{irrev}/dt$$

$$\eta_{is} = \frac{h_a - h_b}{h_a - h_{b'}}$$

$$\eta_{is} = \frac{h_{d'} - h_c}{h_d - h_c}$$



Entropy $S = k\ln(W)$

Entropy is related to thermodynamic probability

GEB

Geothermal Energy Capacity Building in Egypt (GEB)

Thermal Engineering for Geothermal Energy

Co-funded by the
Erasmus+ Programme
of the European Union

Additional

Carnot cycle

Process 1-2:

Isentropic
Heat $|Q_{12}|=0$

Process 2-3:

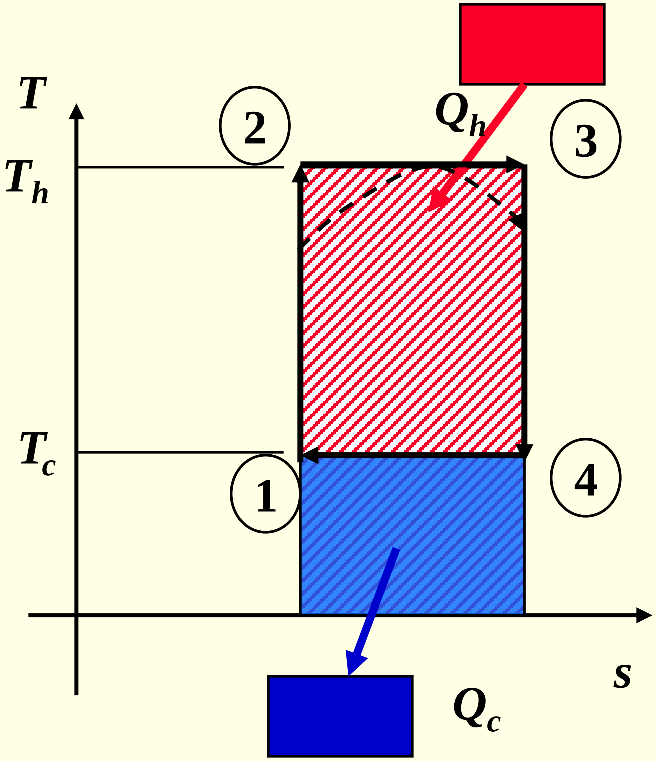
Isothermal heating ($T=T_h$)
Added heat $|Q_h|=T_h\ (s_3-s_2)$

Process 3-4:

Isentropic
Heat $|Q_{34}|=0$

Process 4-1:

Isothermal cooling ($T=T_c$)
Chaleur rejetée $|Q_c|=T_c\ (s_4-s_1)$



$$\eta\ Carnot = 1 - |Q_c/Q_h| = 1 - T_c/T_h$$

Additional

Stirling cycle

Process 1-2:

Isochoric heating
Regenerated heat $|Q_{12}|=u_2-u_1$

Process 2-3:

Isothermal heating ($T=T_h$)
Added heat $|Q_h|=T_h\ (s_3-s_2)$

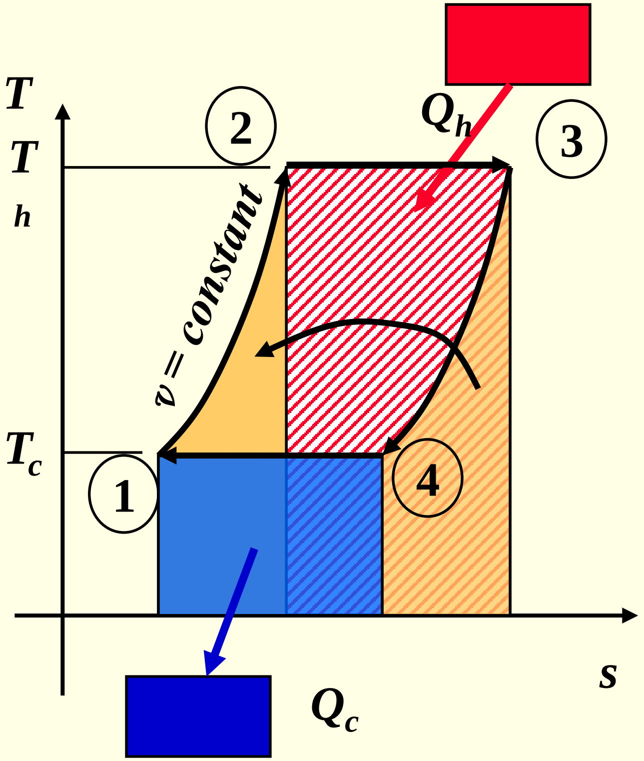
Process 3-4:

Isochoric cooling
Regenerated heat $|Q_{34}|=u_3-u_4=|Q_{12}|$

Process 4-1:

Isothermal cooling ($T=T_c$)
Chaleur rejetée $|Q_c|=T_c\ (s_4-s_1)$

NB $s_4-s_1= R\ln(v_4/v_1) = R\ln(v_3/v_2) = s_3 - s_2$



$$\eta\ \text{Sterling} = 1 - |Q_c/Q_h| = 1 - T_c/T_h$$
$$= \eta\ \text{Carnot}$$

Additional

Ericsson cycle

Process 1-2:

Isobaric heating
Regenerated heat $|Q_{12}|=h_2-h_1$

Process 2-3:

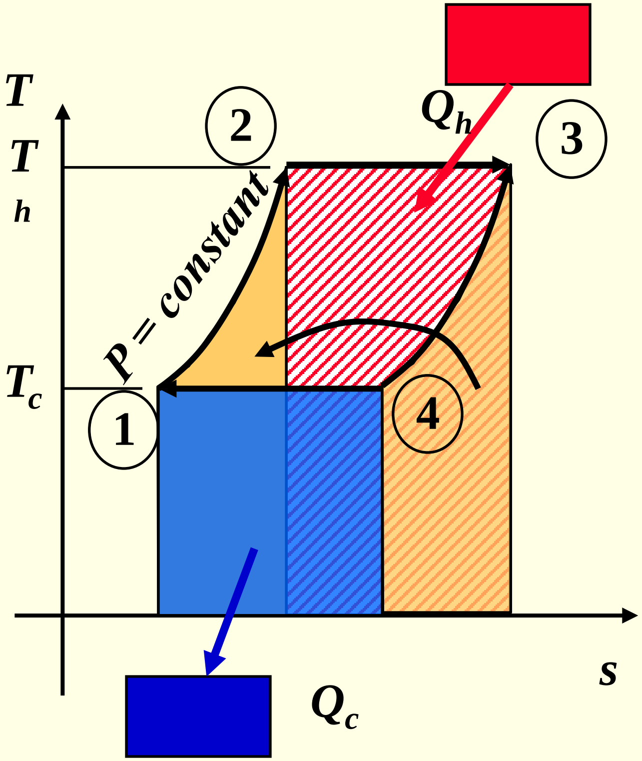
Isothermal heating ($T=T_h$)
Added heat $|Q_h|=T_h\ (s_3-s_2)$

Process 3-4:

Isobaric cooling
Regenerated heat $|Q_{34}|=h_3-h_4=|Q_{12}|$

Process 4-1:

Isothermal cooling ($T=T_c$)
Rejected heat $|Q_c|=T_c\ (s_4-s_1)$



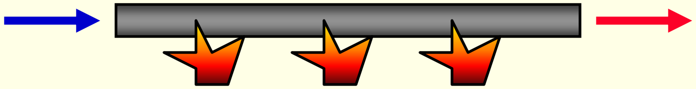
$$\eta\ Ericsson = 1 - |Q_c/Q_h| = 1 - T_c/T_h = \eta\ Carnot$$

NB $s_4-s_1= -R\ ln\ (P_4/P_1) = -R\ ln\ (P_3/P_2) = s_3 - s_2$

Rankine cycle

To imitate Carnot, we need an isothermal heating: very difficult??

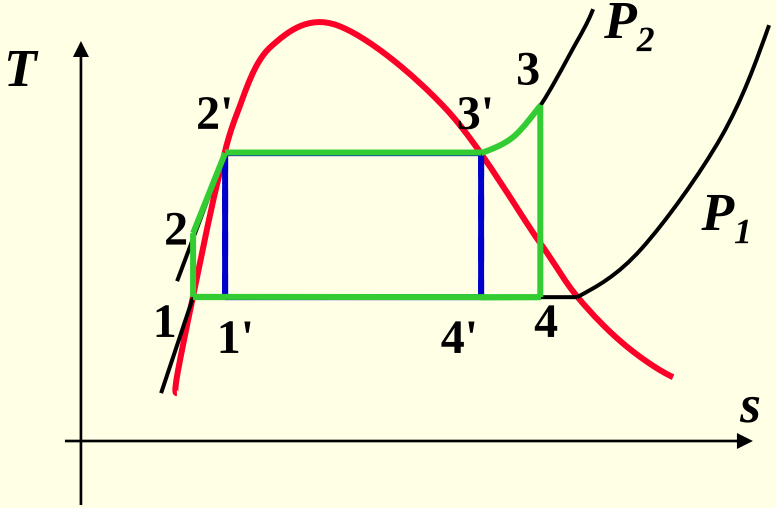
Isobaric heating: Easy !



How about the isobaric heating of a Liquid – vapor mixture

Problem for the compression of a Liquid – vapor mixture

Problem for the expansion of a Liquid – vapor mixture



Rankine cycle: 1-2-3-4

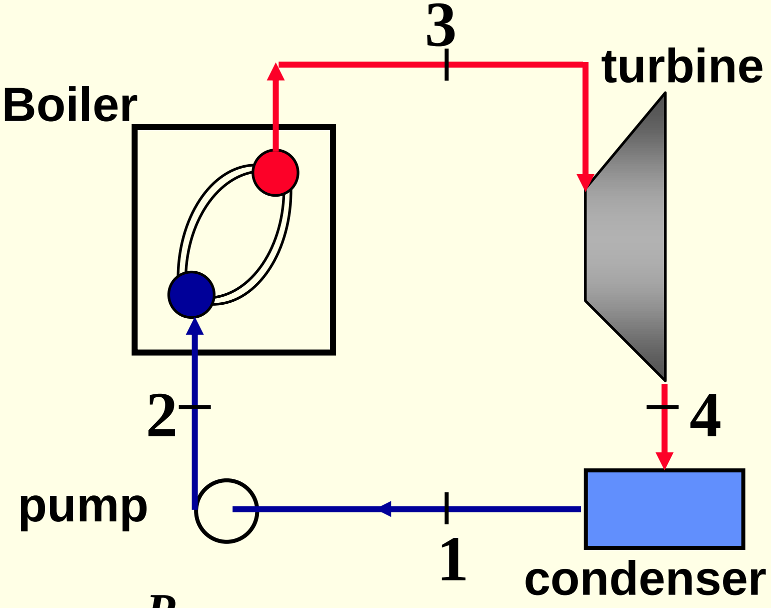
Cycle Parameters

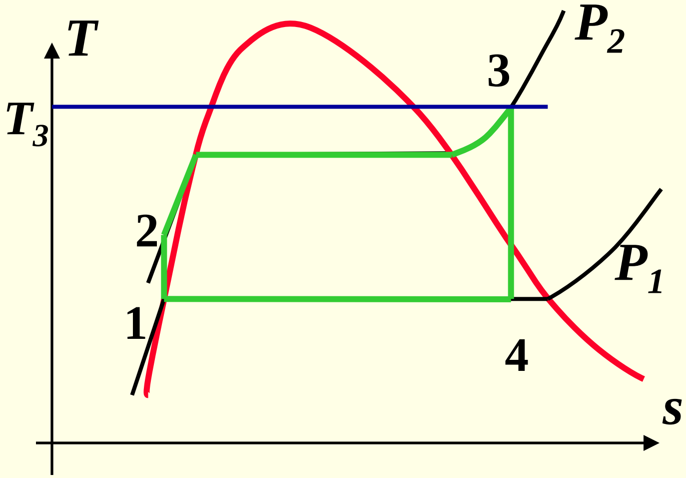
Principle parameters of the cycle:

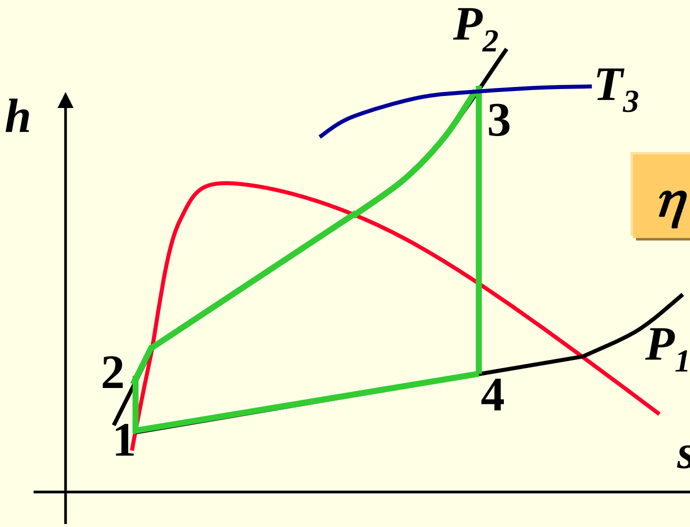
P_1 : Condenser pressure

P_2 : Boiler pressure

T_3 : Temperature at exit of boiler

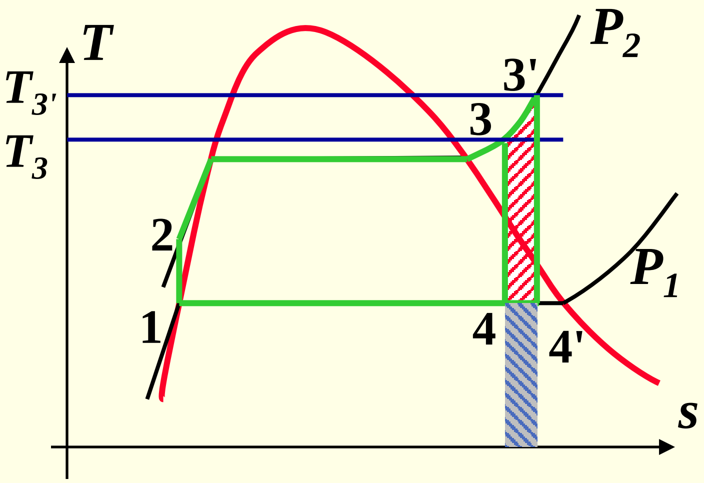




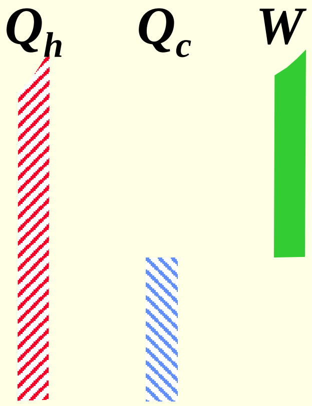


$$\eta = (h_3 - h_4)/(h_3 - h_2)$$

Effect of boiler exit temperature T_3



Increase of :

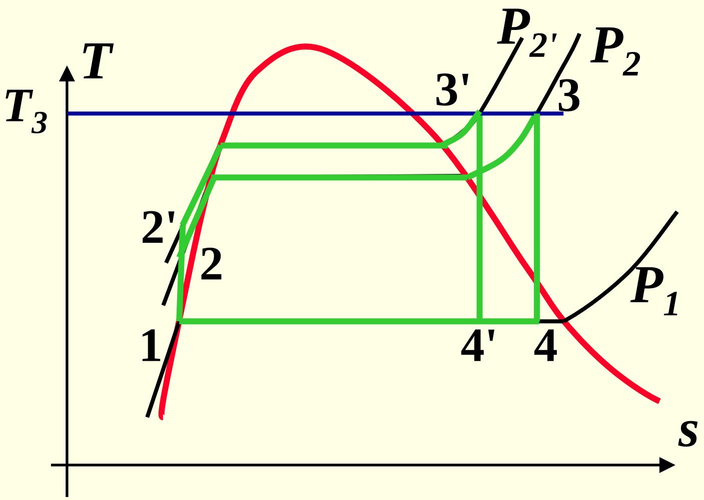


By increasing T_3:
- η increases ☺
- x_4 increases ☺

Average T_h increases

Upper limit of T_3:
Dictated by:
- **Turbine cooling**
- **Metallurgical considerations**

Effect of boiler pressure P_2

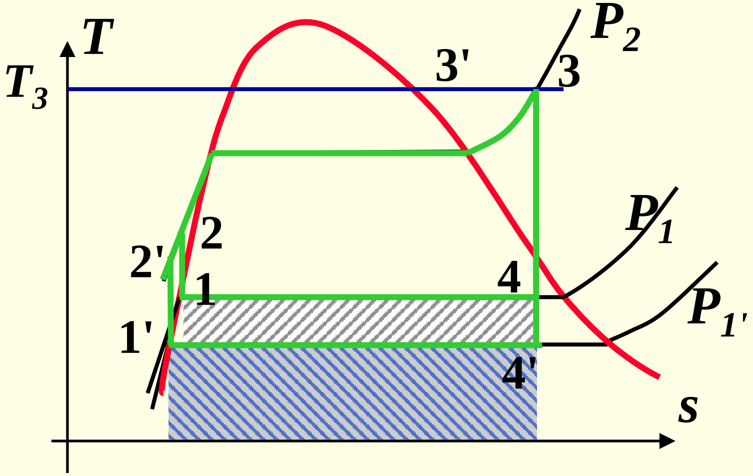


Increase of P_2:

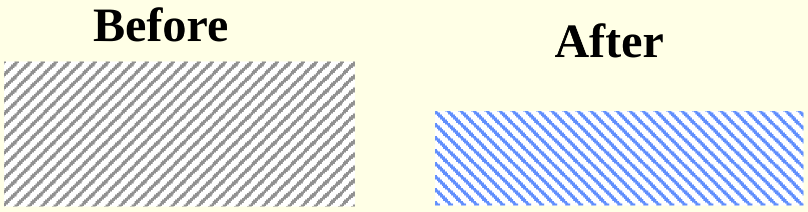
- η increases ☺
- x_4 decreases ☹

Adding heat at ~ constant T_h

Effect of condenser pressure



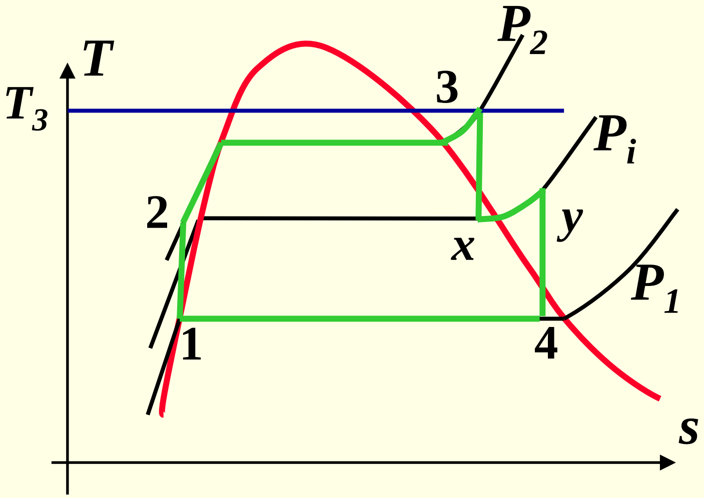
Rejected heat

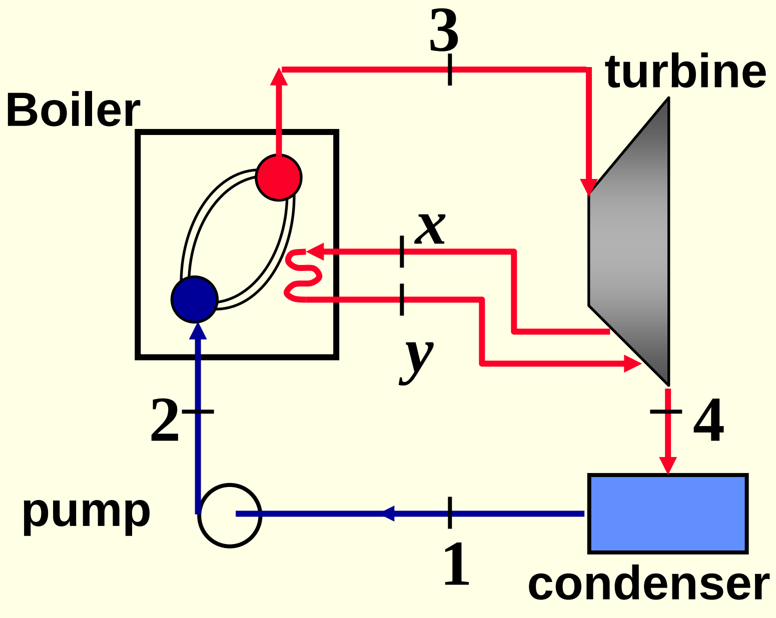


Same added heat
Less rejected geat

Decreasing P_1:
- η **increases** ☺
- x_4 **decreases** ☹

Reheat cycle





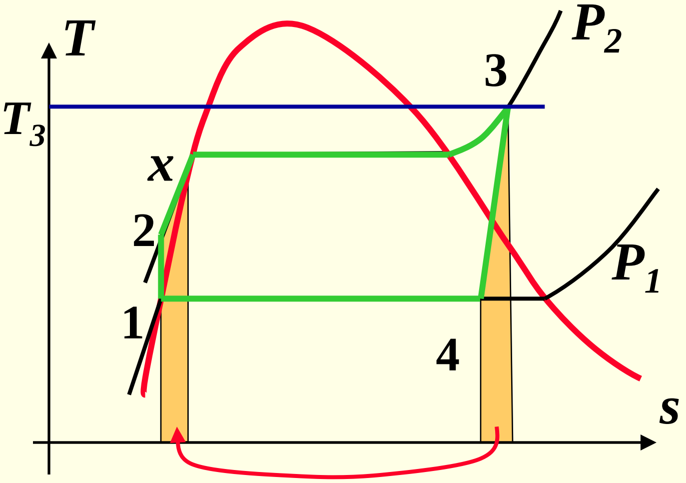
By reheating at P_i:

$\overline{T}$ increases

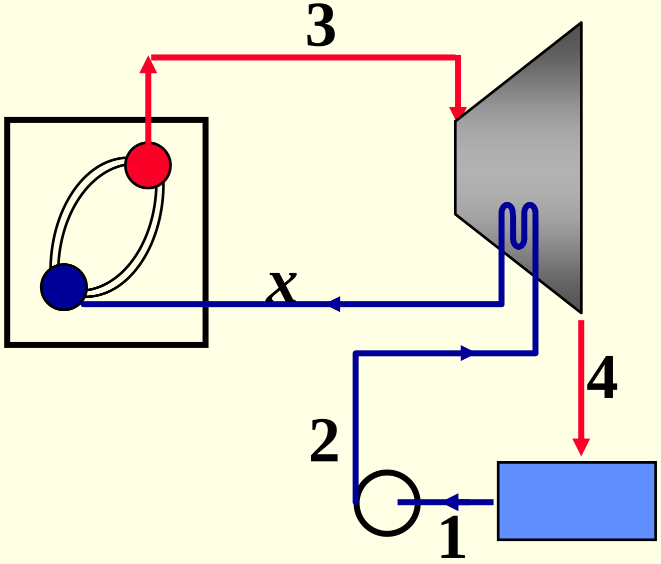
Reheating at P_i:

- η increases ☺
- x_4 increases ☺

Regenerative cycle I



Regenerated heat



By reheating feed water by the vapor in turbine:

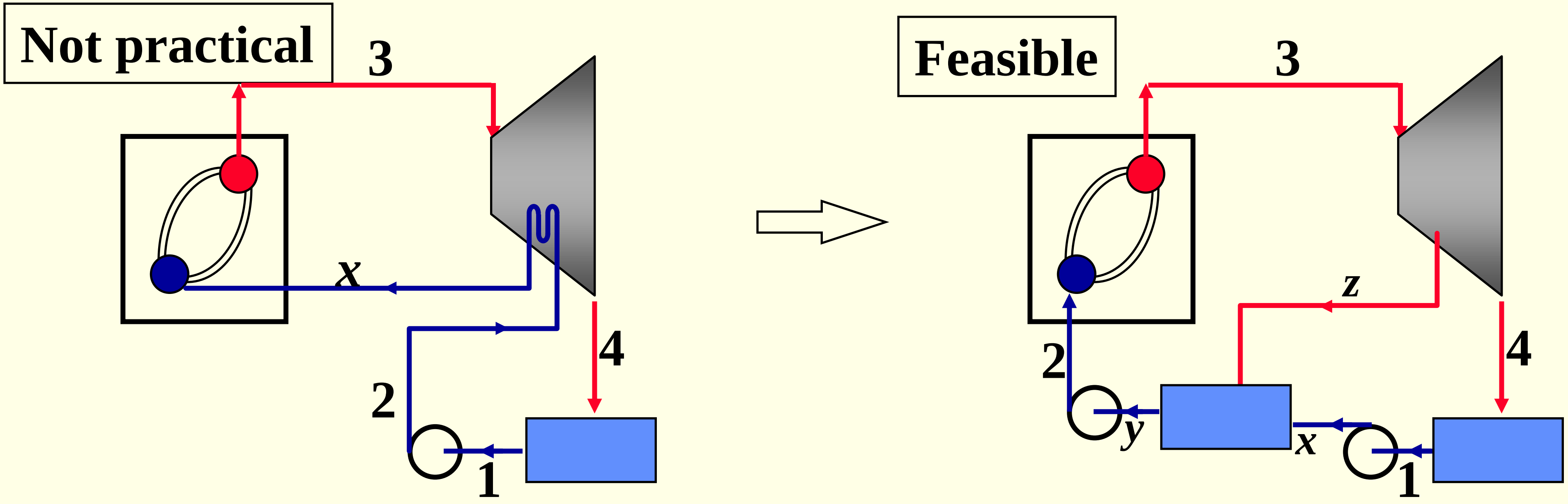
$\overline{T}$ increases

Approaches Sterling/Ericson

Regeneration I:

- **η increases ☺**
- **x_4 decreases ☹**

Regenerative cycle II



Not practical

Feasible

Feed water is heated by:

Part **of the energy of**
All the vapor

All **the energy of**
Part **of the vapor**

Regeneration II:

- η **increases ☺**
- x_4 **constant 😐**

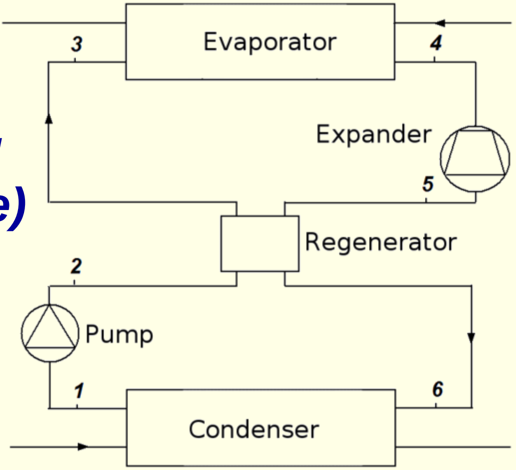
Organic Rankine cycle

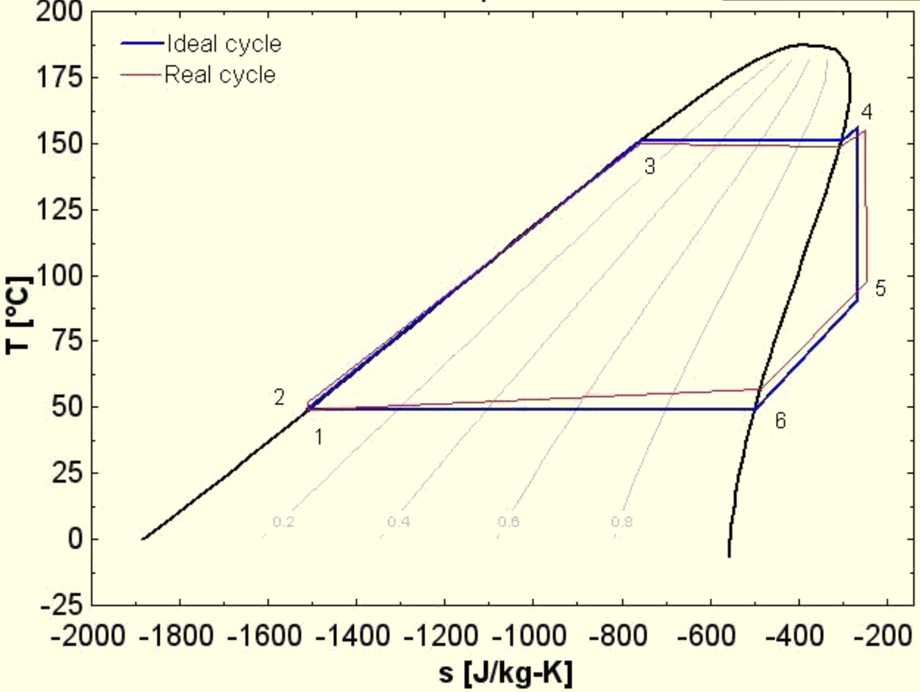
Geothermal heat sources are at low temperatures $T_g \sim 100^oC - 250^oC$

Superheat not suitable: $T_3 \sim T_{sat}$

Water not suitable: dryness $x_4 << 1$

Use organic working fluid (e.g. isopentane) and probably regeneration





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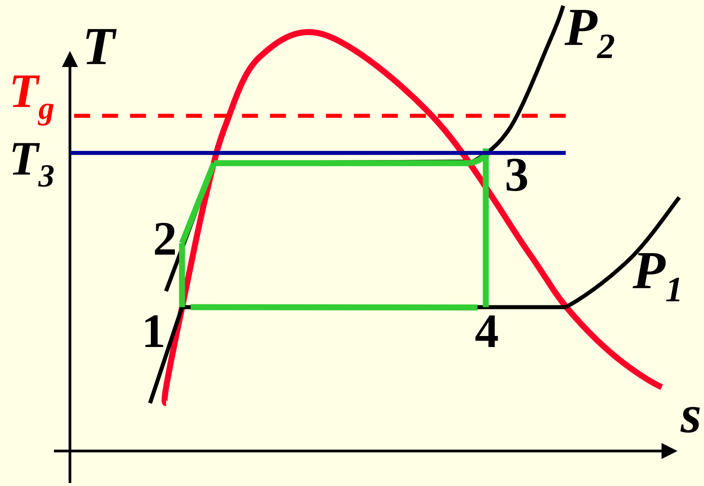
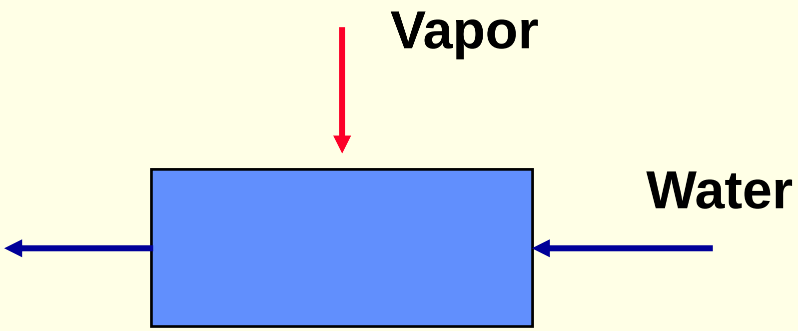


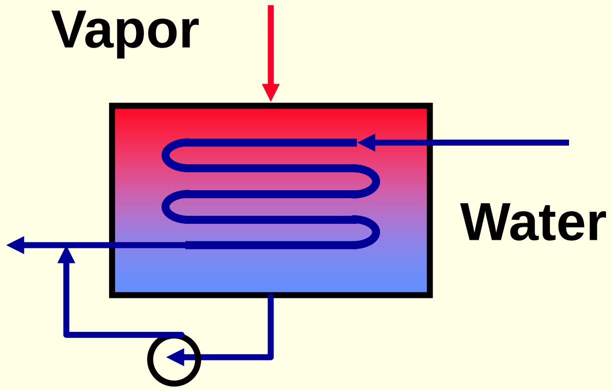
Additional

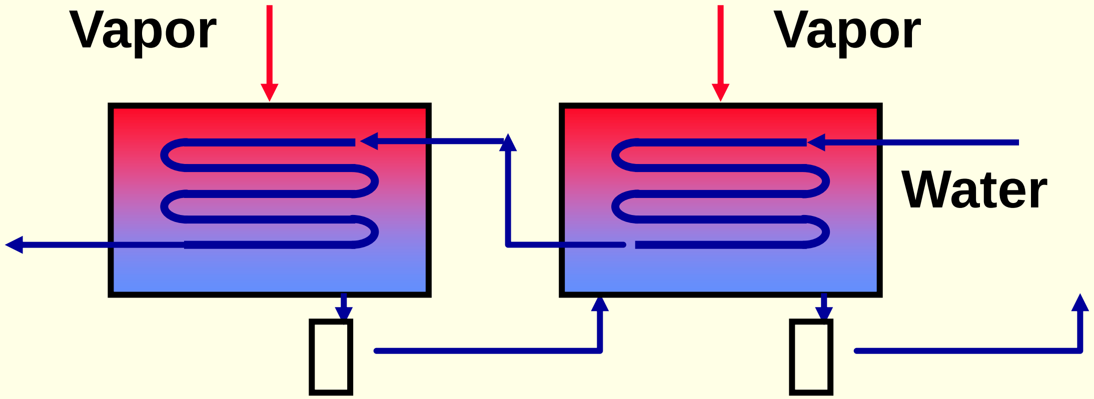
Feed water heaters

Open feed water heate



Closed feed water heater





Additional

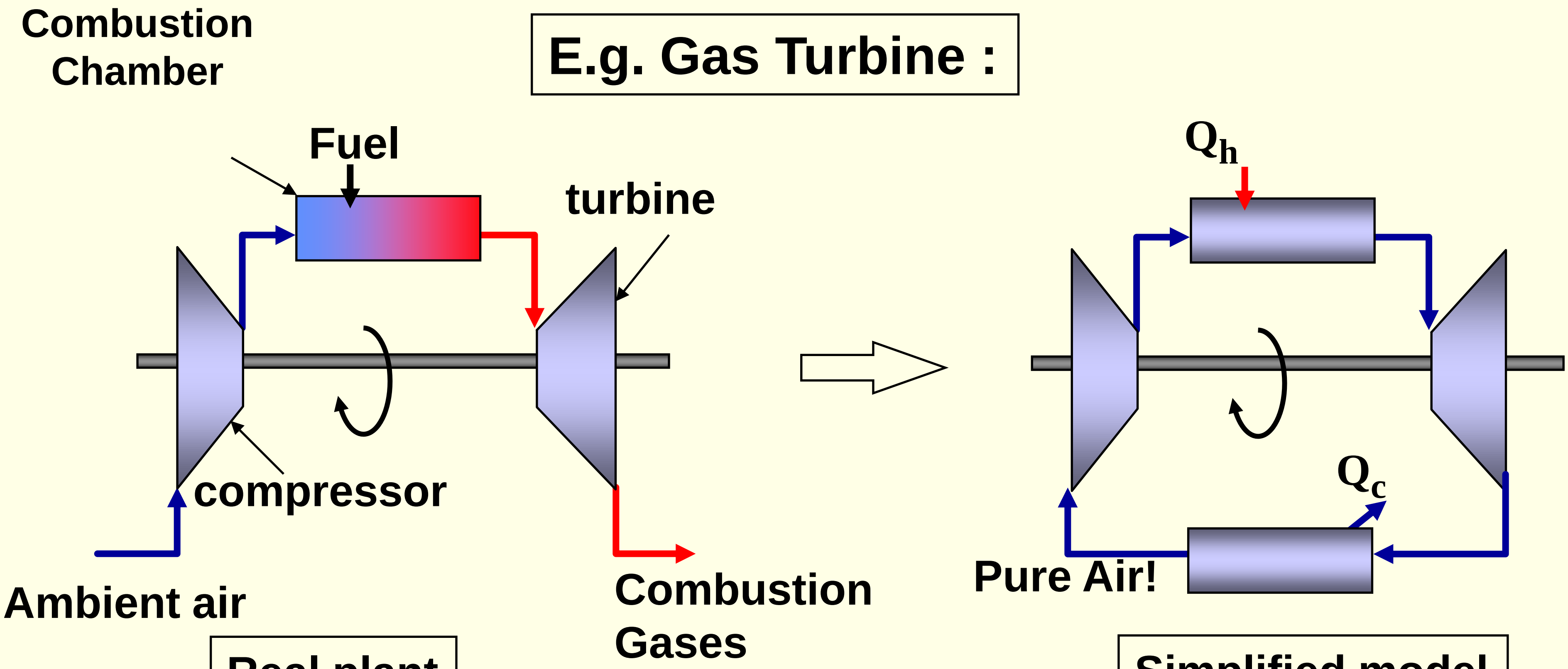
Deviations of real cycle

- **Loss of pressure in boiler**
- **Loss of pressure in condenser**
- **Isentropic efficiency of turbine**
- **…**

<u>Internal</u> combustion engines (ICE)

Heat is added by
<u>combustion of working fluid</u>
(air – fuel mixture)

E.g. Gas Turbine :



Real plant

Simplified model

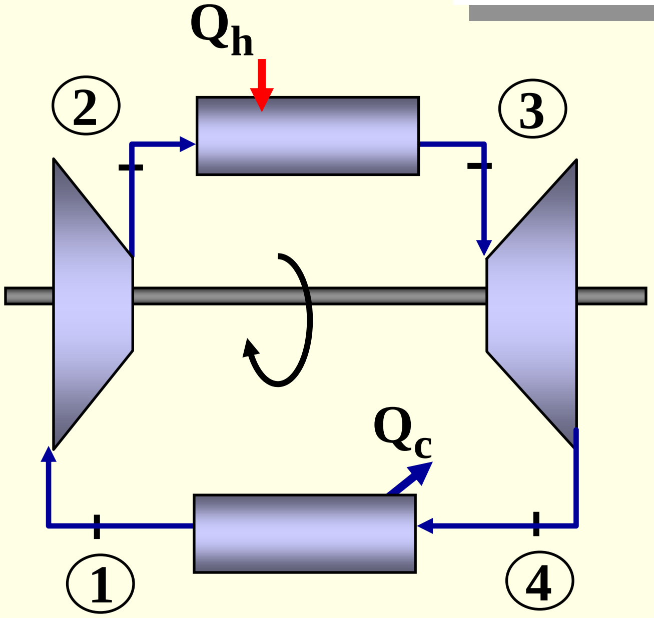
Hypotheses

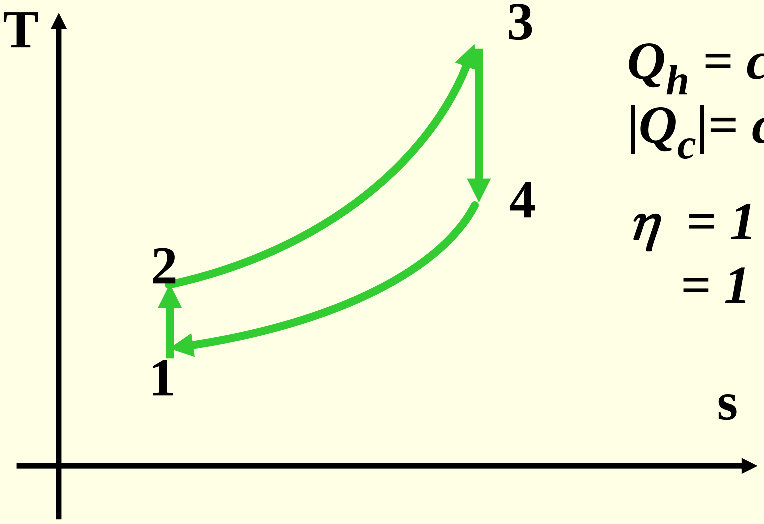
- **All processes are reversible**

- **Fluid: Gas having a constant chemical composition**
  - **Heat is added by an external source**
  - **Closed Cycle: flue gases are cooled and reused**

- **Perfect Gas**
  - $P\ v = R\ T$
  - **Constant thermal capacities**

Joule (Brayton) Cycle





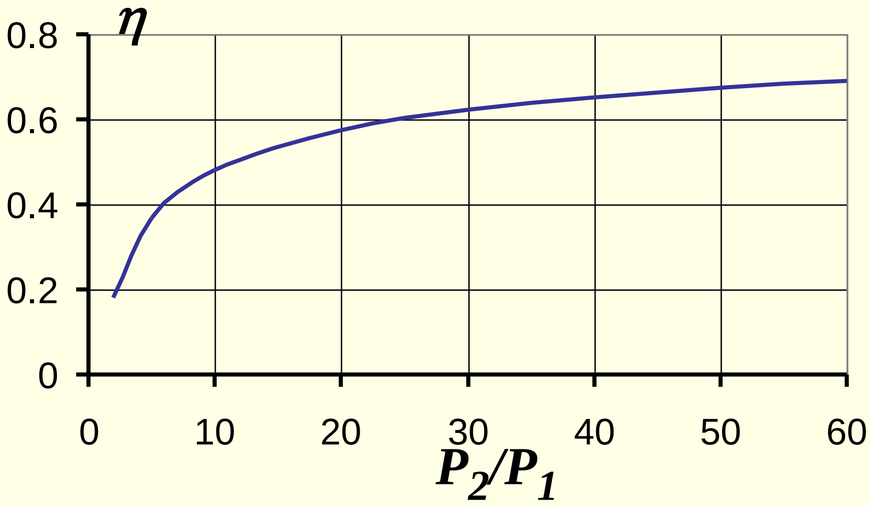
$Q_h = c_p\,(T_3 - T_2)$
$|Q_c| = c_p\,(T_4 - T_1)$

$$\eta = 1 - |Q_c| / Q_h = 1 - (T_4 - T_1) / (T_3 - T_2)$$

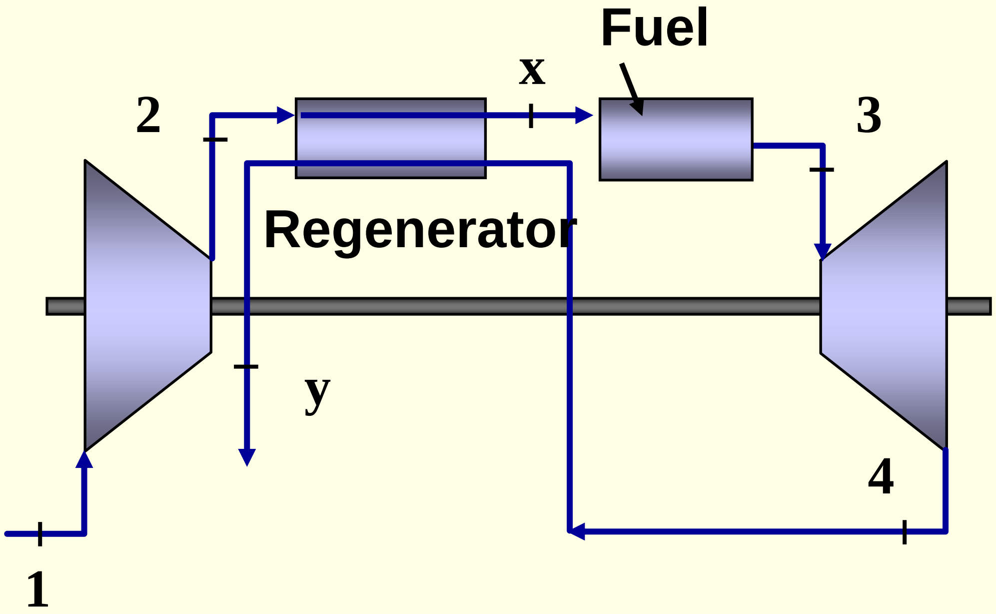
In the compressor: $T_2/T_1 = (P_2/P_1)^{(\gamma-1)/\gamma}$
In the turbine: $T_3/T_4 = (P_2/P_1)^{(\gamma-1)/\gamma}$

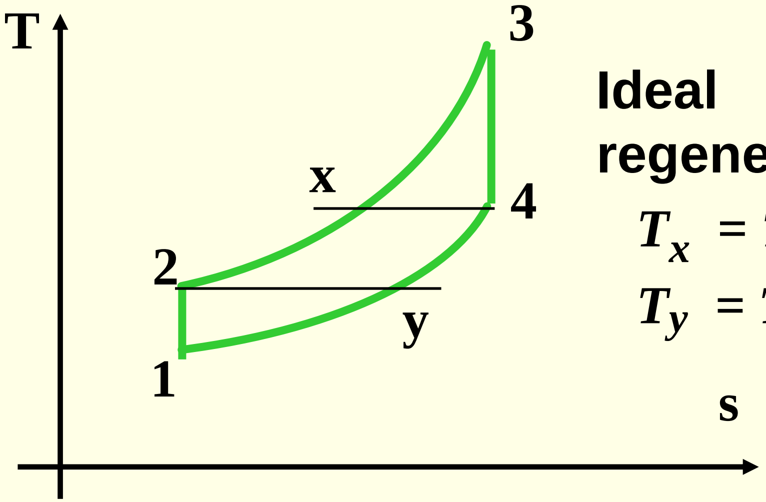
⇨ $$\boxed{T_3/T_4 = T_2/T_1 = (T_3 - T_2) / (T_4 - T_1)}$$

⇨ $$\boxed{\eta = 1 - T_1 / T_2 = 1 - 1/ (P_2/P_1)^{(\gamma-1)/\gamma}}$$



Regenerative Joule cycle





Ideal regeneration

$T_x = T_4$

$T_y = T_2$

$$Q_h = c_p\,(T_3 - T_x) = c_p\,(T_3 - T_4) = W_{turb}$$

$$|Q_c| = c_p\,(T_y - T_1) = c_p\,(T_2 - T_1) = |W_{comp}|$$

$$\eta = 1 - (T_2 - T_1)\,/\,(T_3 - T_4)$$

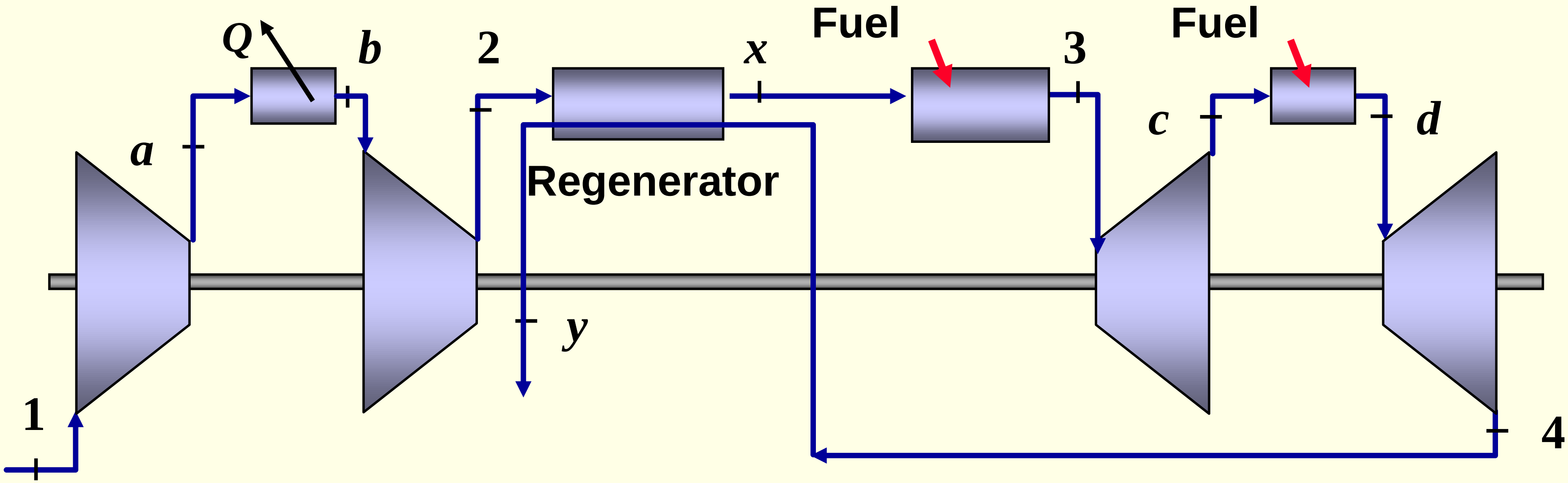
But:

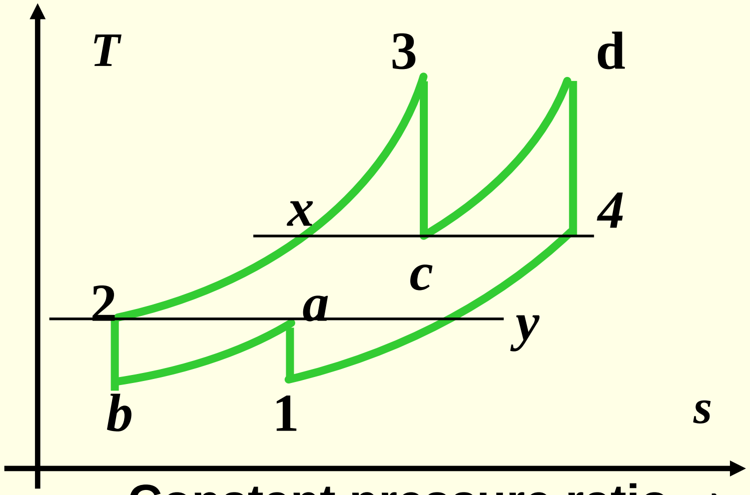
$$\frac{T_2 - T_1}{T_3 - T_4} = \frac{T_1\cancel{(T_2/T_1-1)}}{T_4\cancel{(T_3/T_4-1)}} = \frac{T_1\,T_3}{T_3\,T_4} = \frac{T_1}{T_3}\left[\frac{P_2}{P_1}\right]^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \quad \boxed{\eta = 1 - (T_1/T_3)\,(P_2/P_1)^{(\gamma-1)/\gamma}}$$

Additional

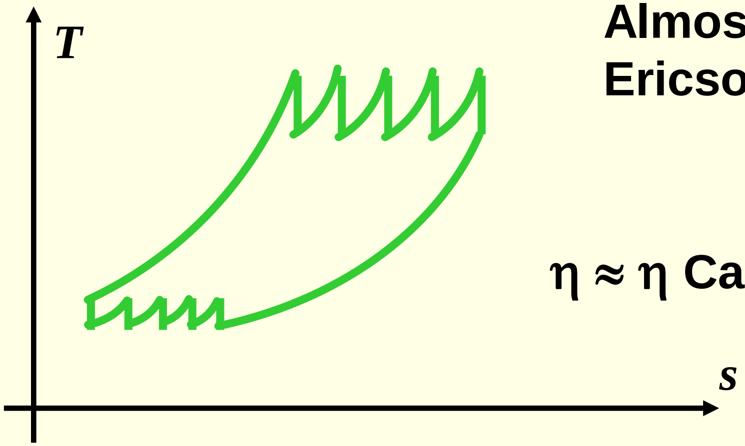
Regenerative cycle with Multi stage reheat *Joule*





Constant pressure ratio ⇒ Max η

N-Stages ⇒



Almost Ericson

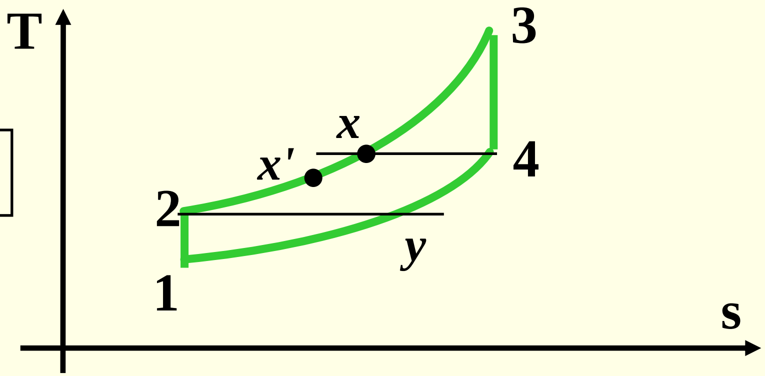
η ≈ η Carnot

Additional

Deviations of real cycle

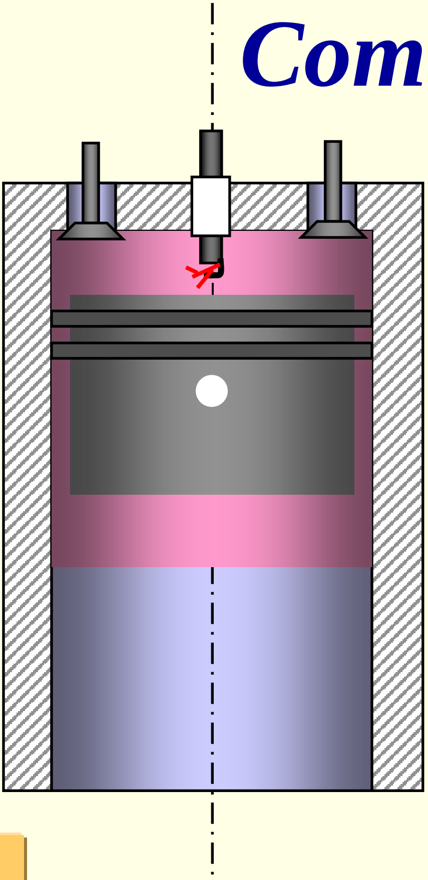
- Variation of gas thermal capacity $c_p(T)$
- Variable chemical composition (Combustion, Dissociation CO, NO, …)
- Efficacy of regenerator
- Pressure drops (combustion chamber, regenerator, ..)
- Isentropic Efficiencies of turbine and compressor

$$\eta_{htr} = (h_{x'} - h_2)/(h_x - h_2)$$



Additional

Combustion engines



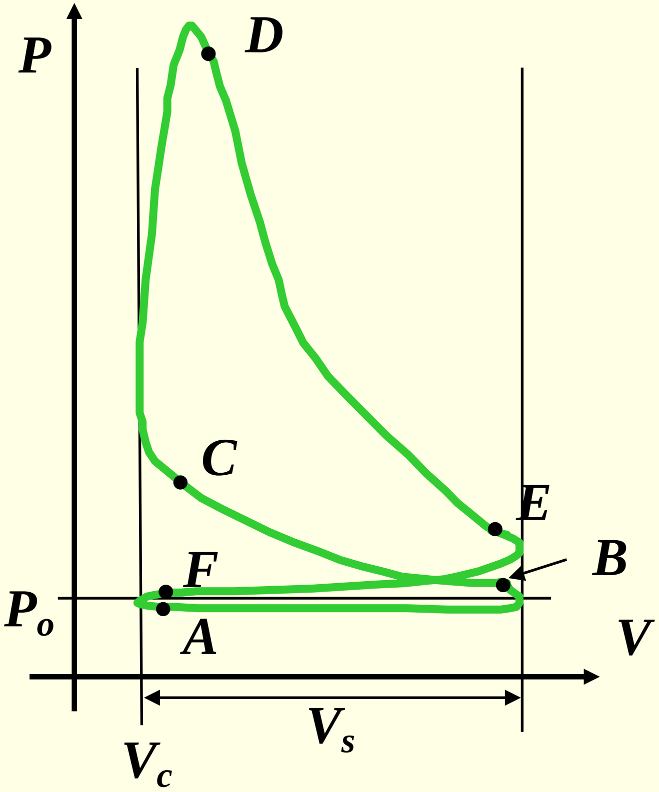
Spark ignition

Compression ignition

At intake: pure air

At end of compression:

- Fuel injection
- Ignition by contact of hot air

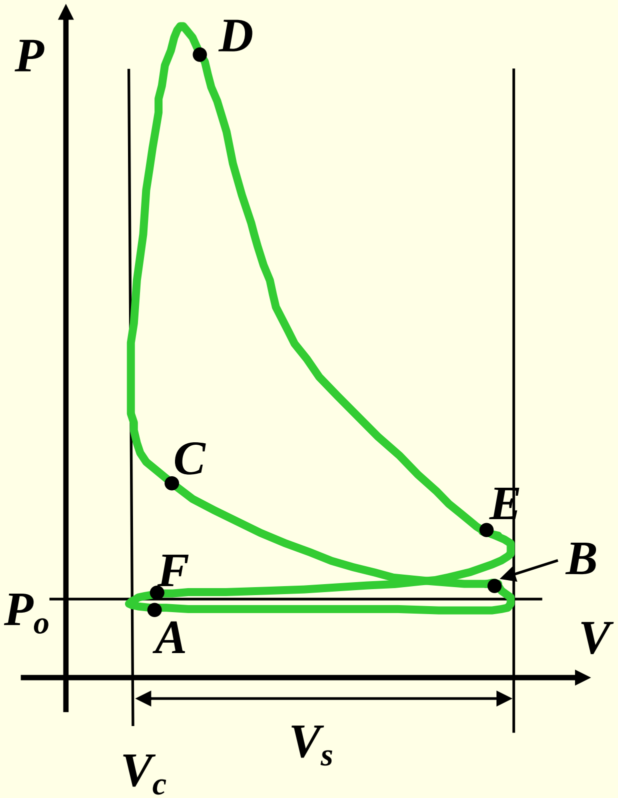


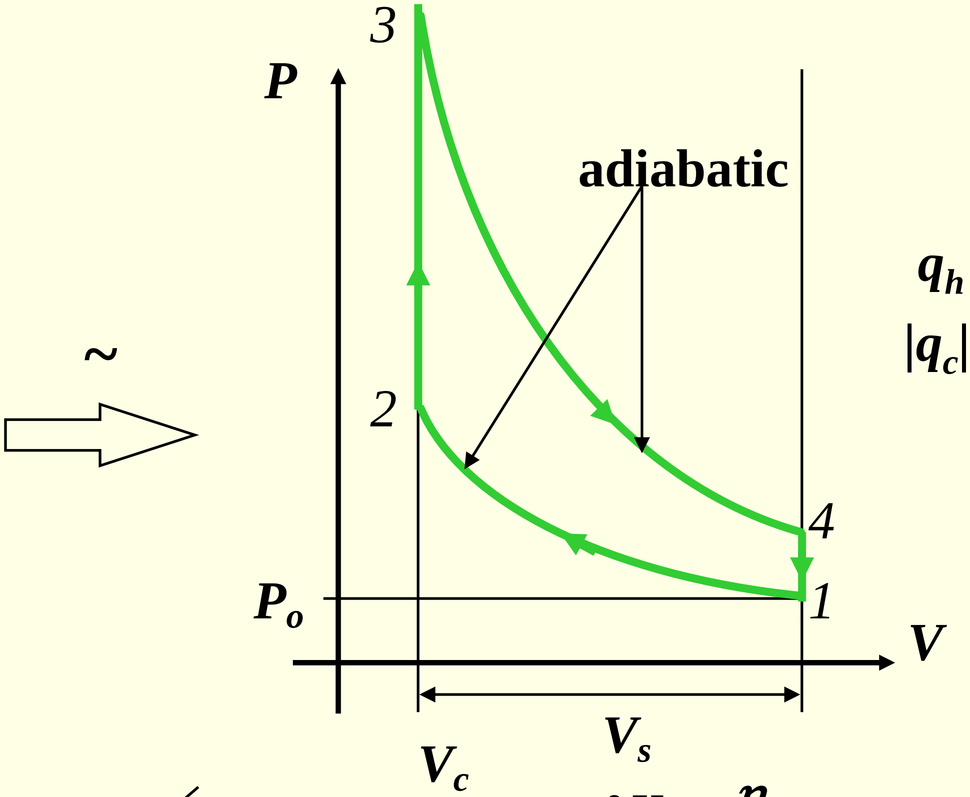
Compression ratio

$r = (V_s + V_c) / V_c$

Additional

Otto Cycle





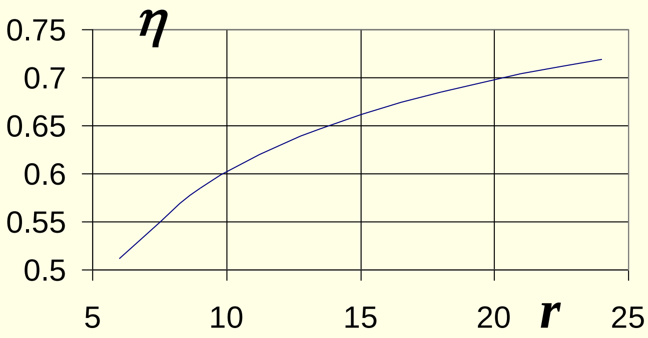
$$\boxed{\begin{aligned} r &= V_1/V_2 \\ &= V_4/V_3 \end{aligned}}$$

$$q_h = c_v\,(T_3 - T_2)$$

$$|q_c| = c_v\,(T_4 - T_1)$$

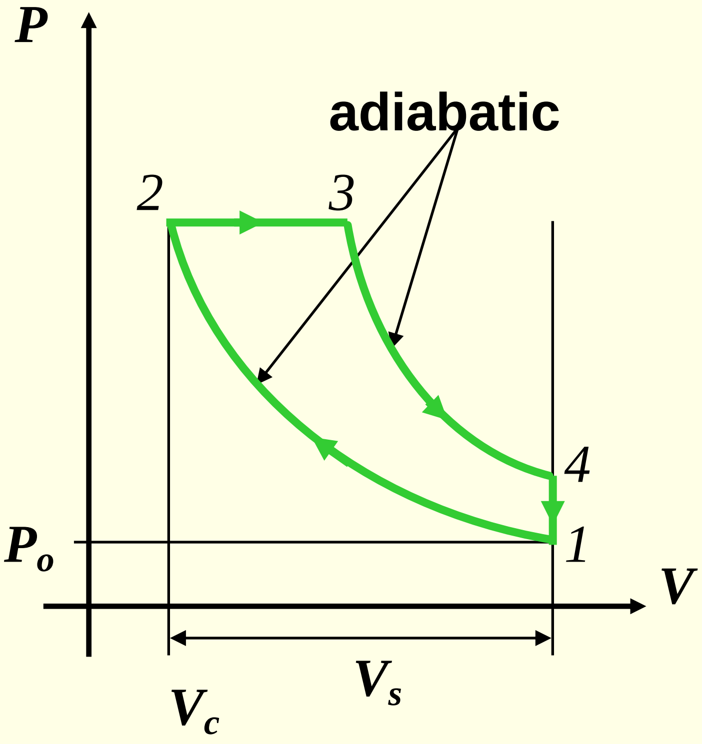
$$\eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \quad = 1 - \frac{T_1\cancel{(T_4 / T_1 - 1)}}{T_2\cancel{(T_3 / T_2 - 1)}}$$

$$\boxed{\eta = 1 - 1/r^{(\gamma-1)}}$$



Additional

Diesel cycle



$$q_h = c_P\,(T_3 - T_2)$$
$$|q_c| = c_v\,(T_4 - T_1)$$

$$\Rightarrow \boxed{\eta = 1 - \frac{c_v(T_4 - T_1)}{c_P(T_3 - T_2)}}$$

Definition: $r_v = V_3 / V_2$

$$T_4/T_3 = (V_3/V_4)^{(\gamma-1)} = (V_2V_3/V_4V_2)^{(\gamma-1)} = (r_v/r)^{(\gamma-1)}$$

$$T_3/T_2 = V_3/V_2 = r_v$$

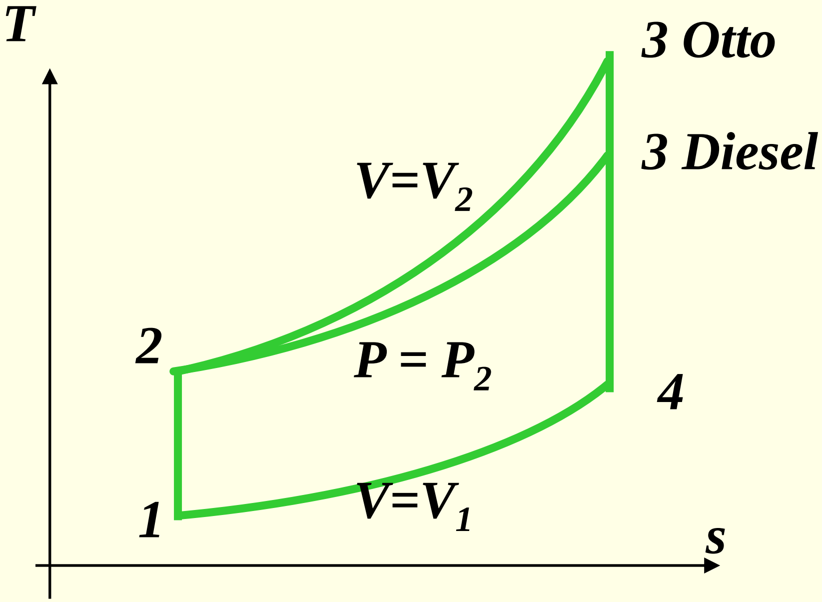
$$T_2/T_1 = (V_2/V_1)^{(\gamma-1)} = r^{(\gamma-1)}$$

$$\Rightarrow \quad T_4/T_1 = r_v^{\gamma}$$

$$\eta = 1 - \frac{T_1(T_4/T_1 - 1)}{\gamma T_2(T_{3b}/T_2 - 1)} \quad \Rightarrow \quad \boxed{\eta = 1 - \frac{(r_v^{\gamma} - 1)}{r^{(\gamma-1)}\gamma(r_v - 1)}}$$

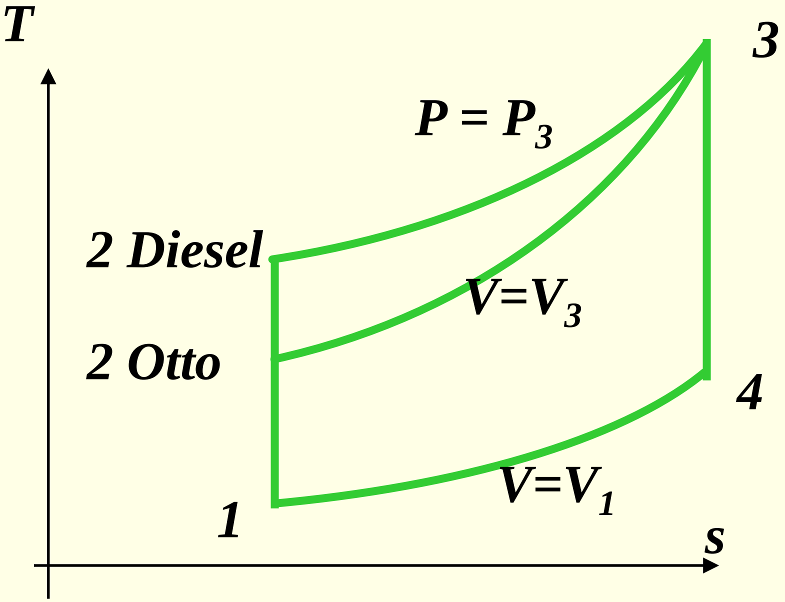
Additional

Otto - Diesel



Same Compression ratio *r*

η Otto > η Diesel

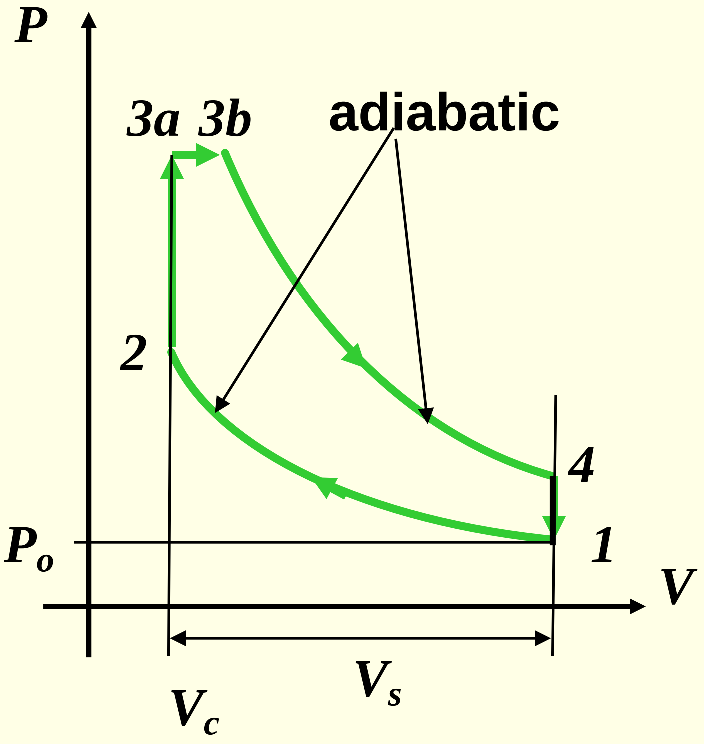


Same T Max. & P Max. of cycle

η Diesel > η Otto

Additional

Dual cycle



Heat addition
- Isochore: 2 – 3a
- Isobar: 3a – 3b

(Fast Diesel motors)

Ignition system

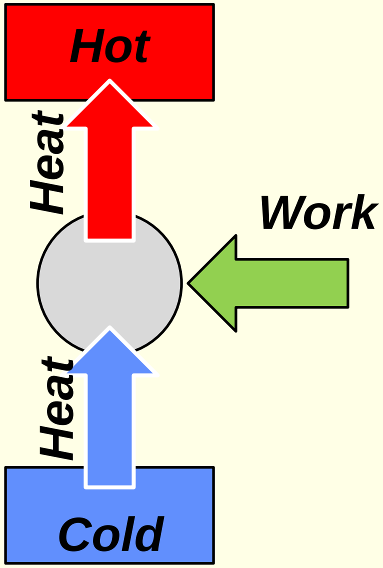
	Spark	Compression
Fuel	**Gasoline (light)**	**Diesel fuel (heavy)**
Ideal cycle	**Otto**	**Diesel or Combined**
Compression ratio	**6 - 10**	**12 - 25**
Efficiency	**30 - 35%**	**40 - 45%**
Power/volume	**30-50 kW/l**	**25-40 kW/l**
Other	**fast response**	**longer life**

Refrigeration and heating cycles

Refrigerators & Heat pumps have in common:

- *They receive work*
- *To transfer heat from a cold space*
- *To heat a hot space*

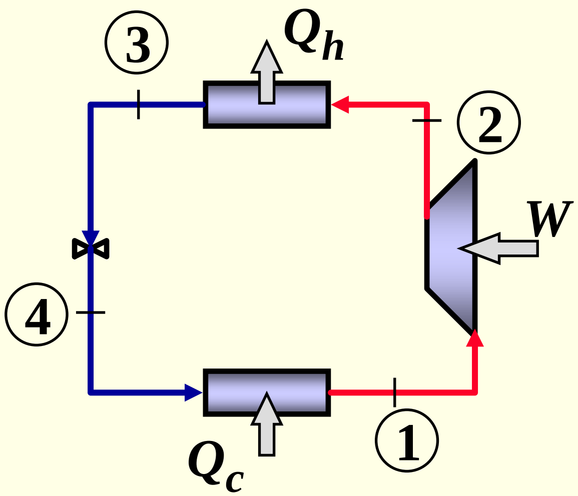
Same cycle may be used for cooling or heating

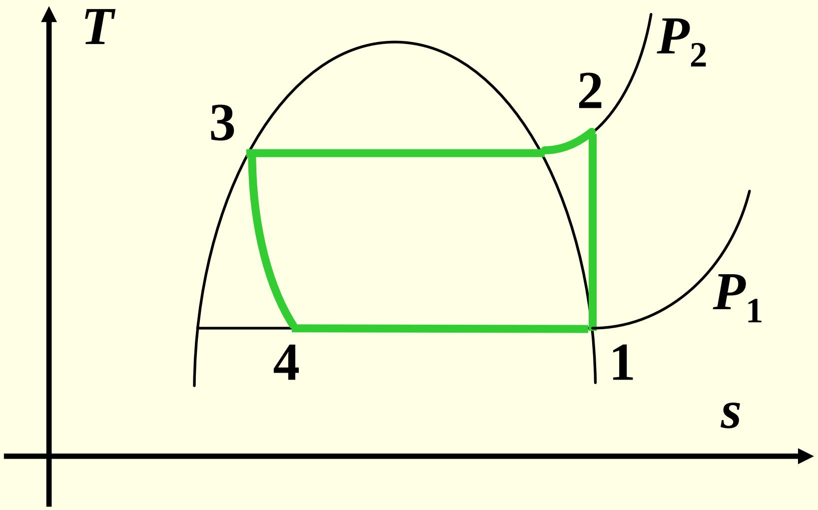


Refrigeration / Heating cycles include (see below):

- *Vapor compression cycle*
- *Absorption cycle*
- *Reversed Joule cycle*

Vapor compression cycle





"Inverted" Rankine Cycle ⇨ Refrigerating effect

Practical Modifications

Process 3-4: throttling (expansion work is negligeable)

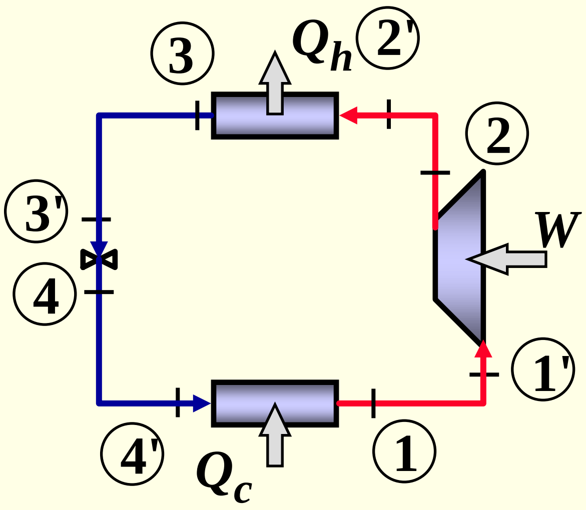
State 1: starts at saturation or superheat
(to preserve the compressor)

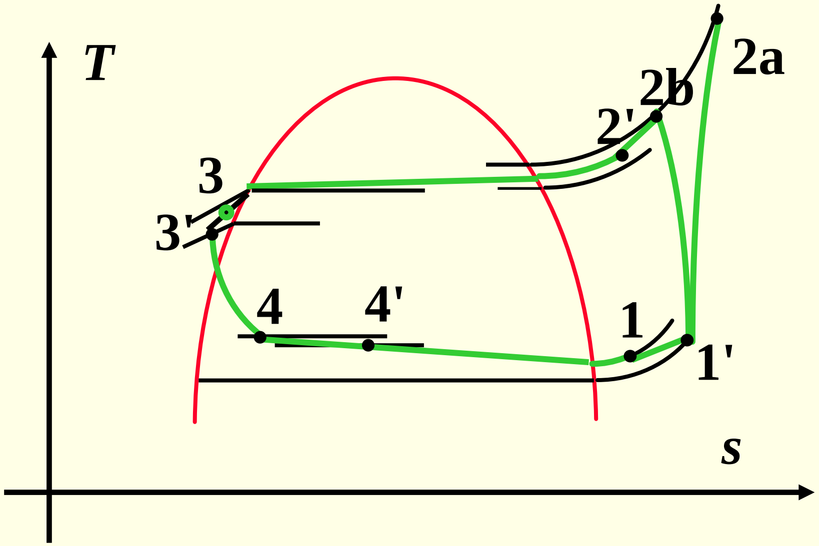
Refrigerants

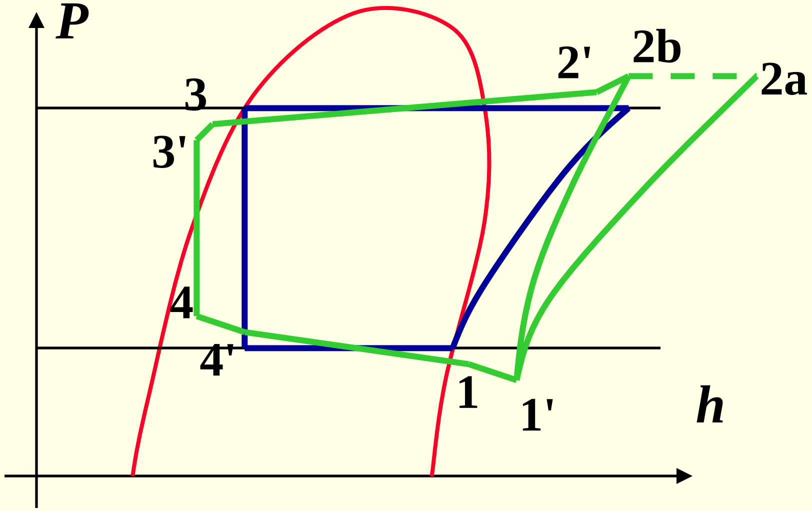
- CFC (ChloroFluroCarbons) gases are banned (Ozone pb)
- HFC (HydroFluroCarbons i.e. no chlore) are replacing
- Gases should have adequate triple and critical points for the application
- No R22, but rather R-410A, R-407C, R-134a

Additional

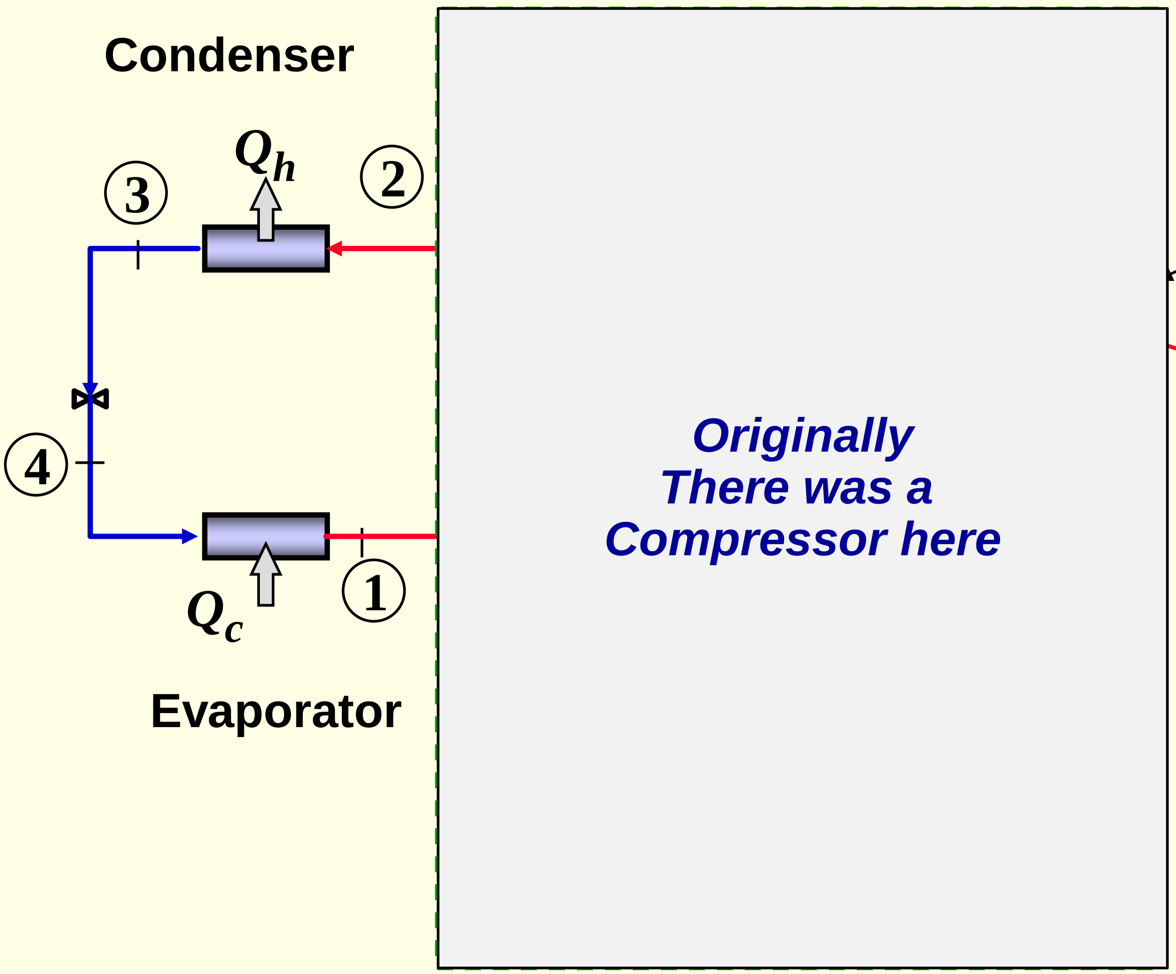
Real Cycle







Absorption cycle



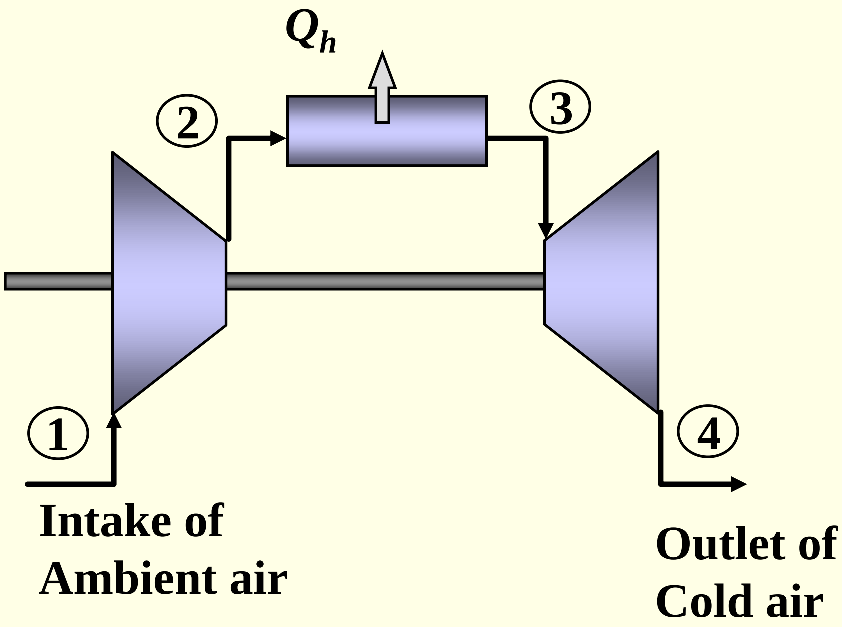
Compressor is replaced by:

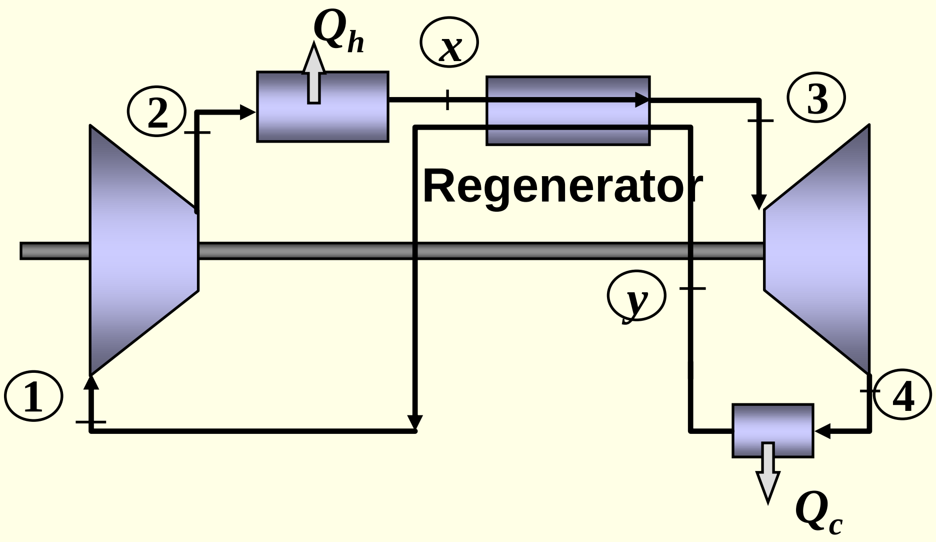
- vapor is absorbed in a liquid
- liquid mixture is pumped (low work)
- vapor is then freed out of liquid

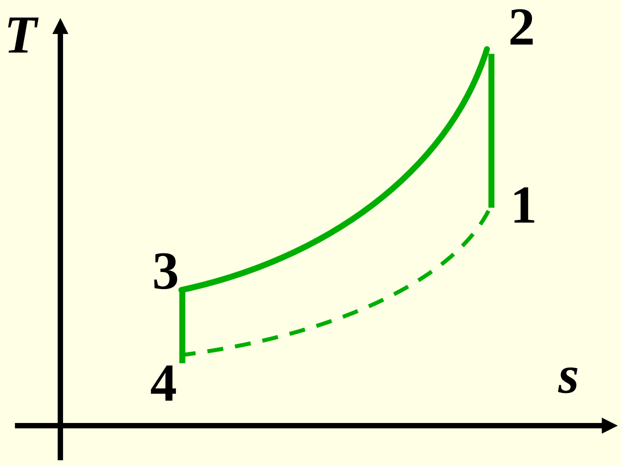
Price paid for very small work:
Heat is added Q_a
At high temperature

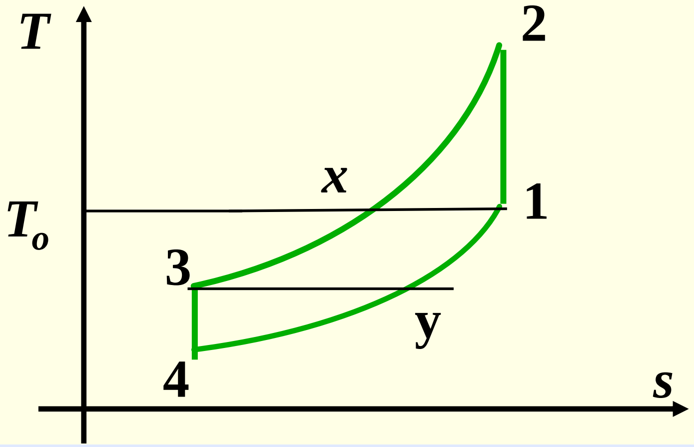
Refrigerant	Solvent
NH_3	H_2O
H_2O	H_2O+CaCl

Reversed Joule cycle



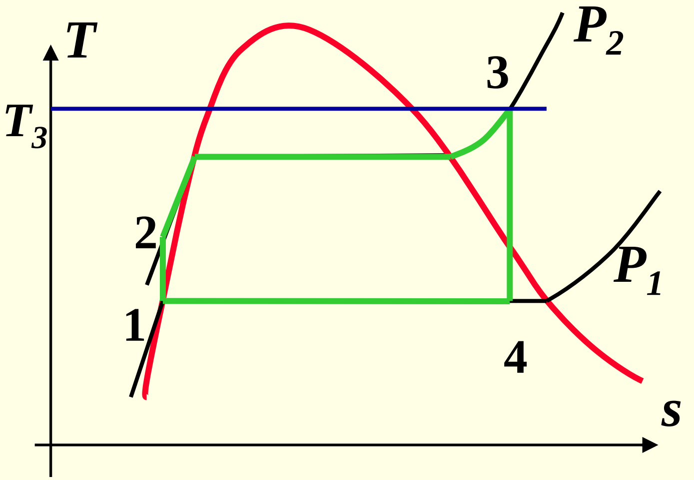




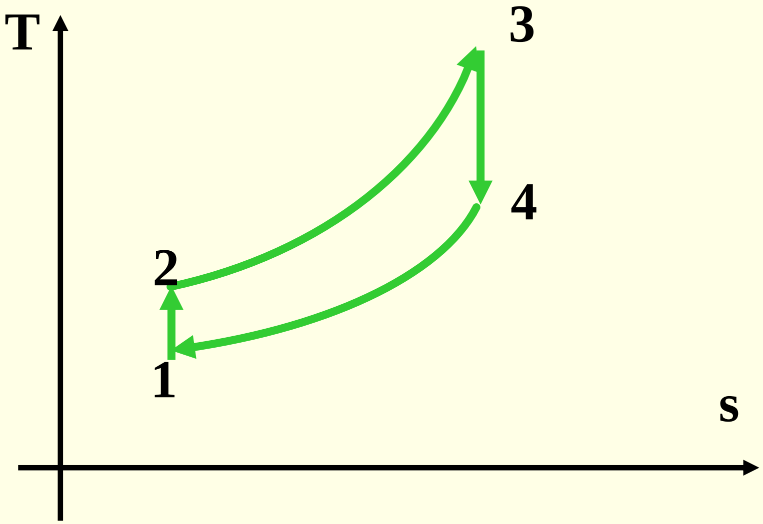


To retain of chapter 8 – A

- ***Sterling** and **Ericson** cycles can be viewed as regenerative Carnot cycle*
- ***Rankine** cycle is the practical implementation of Carnot cycle*
- ***Rankine cycle can be improved by reheat and/or regeneration***

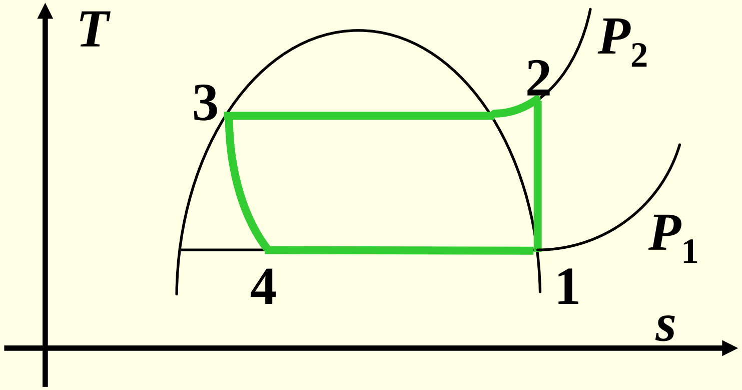


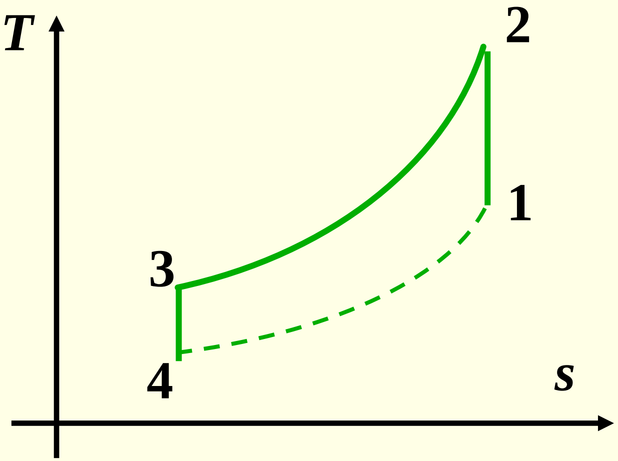
- ***Joule** (Brayton) cycle models gas power plants*
- ***Joule cycle can be improved by reheat and/or regeneration***



To retain of chapter 8 – B

- *Same cycle can be used either for cooling or heating purposes*
  - *Refrigeration cycles are either:*
    - *Vapor compression (inverted Rankine)*
    - *Absorption cooling (vapor compression without compressor)*
    - *Inverted Joule*

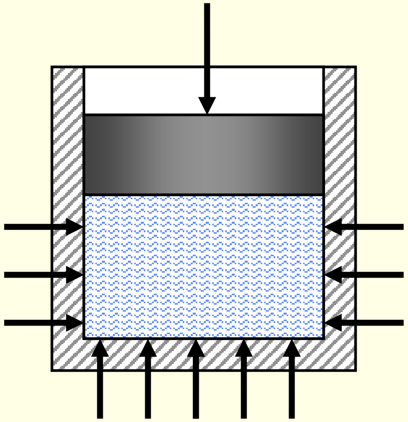




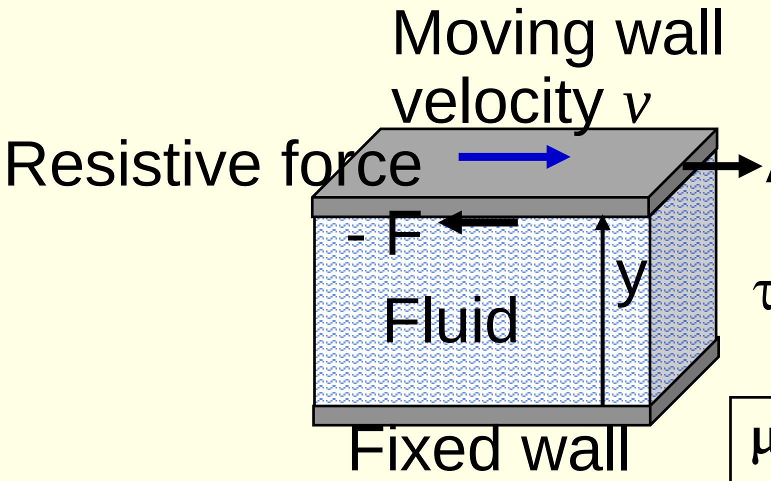
What is a fluid

Matter undergoing continuous deformation under shear stress
Can be a liquid (incompressible) or a gas (compressible)

Stress = External force applied on the surface per unit area
Pressure : Normal component of stress
Shear : Tangential component of stress



Fluid can be at rest under pressure



$$\tau = -F/A = -\mu\,\partial v/\partial y$$

μ: dynamic viscosity
ν=μ/ρ kinematic visc.

Flow description

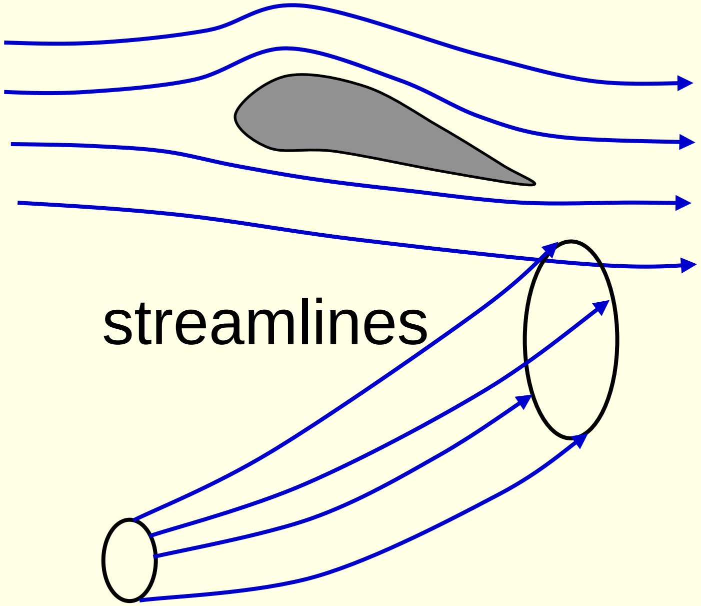
$\mathbf{v}(\mathbf{x},t)$ Velocity at any point $\mathbf{x}$ and time t

Uniform flow: $\mathbf{v}$ is independent of $\mathbf{x}$

Steady flow : $\mathbf{v}$ is independent of t

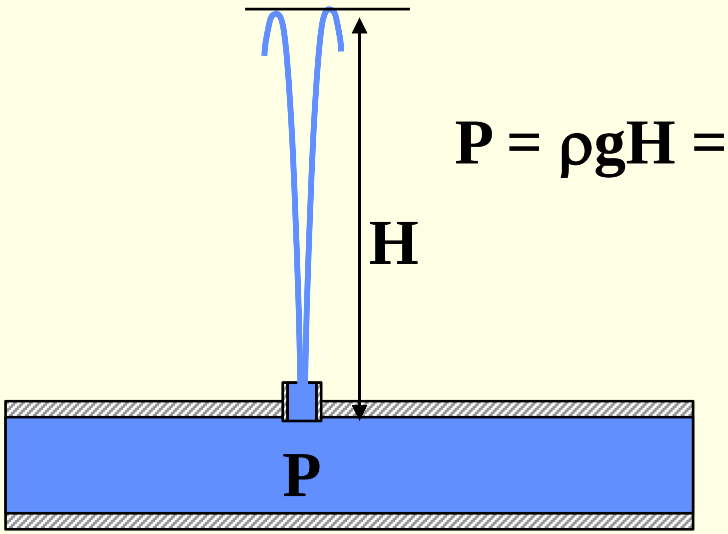
Incompressible flow: ‘No’ changes of density

Compressible flow: ‘appreciable’ changes of density



streamlines

A stream tube



$$P = \rho g H = \gamma H$$

Conservation equations

For any system: **Sources + incoming – outgoing = accumulation**

Mass balance $0 + \dot{m}_{in} - \dot{m}_{out} = dm_s/dt$

$$\dot{m} = \int \rho \mathbf{v} \cdot \mathbf{n} dA \approx \rho v_n A$$

Momentum $\mathbf{F} + \dot{m}_{in}\mathbf{v}_{in} - \dot{m}_{out}\mathbf{v}_{out} = d(m_s\mathbf{v}_s)/dt$

Energy

$$\dot{Q} + \dot{W} + \dot{m}_{in}\left(h + KE + PE\right)_{in} - \dot{m}_{out}\left(h + KE + PE\right)_{out} = \frac{d}{dt}\left[m_s\left(u + KE + PE\right)_s\right]$$

For a steady state flow (SSF): $\boldsymbol{q = \dot{Q}/\dot{m} \quad w = \dot{W}/\dot{m}}$

$\boldsymbol{d(.)/dt = 0; \quad \dot{m}_{in} = \dot{m}_{out} = \dot{m}}$

$$q + w = \Delta\left(h + KE + PE\right)$$

For adiabatic flow of liquids: $q = 0 \quad w = \Delta P_{pump}/\rho \quad \Delta h = \Delta P/\rho + \Delta u$

Bernoulli's eq.:

$$\Delta P_{pump}/\rho - \Delta u = \Delta P/\rho + \Delta v^2/2 + g\Delta z$$

Losses due to friction

$$\Delta P_{pump}/\gamma - h_l = \Delta P/\gamma + \Delta v^2/2g + \Delta z$$

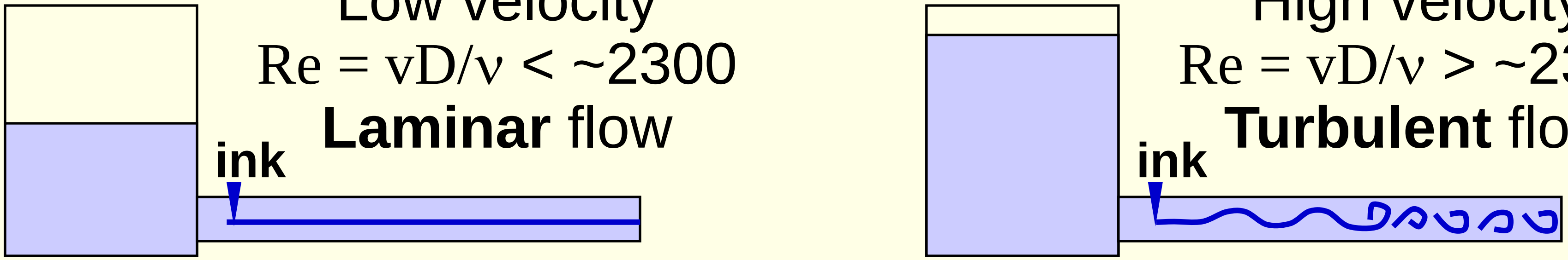
Head lost Pressure head velocity head

Pump power

$\dot{W} = \dot{m}\Delta P_{pump}/r$

$\dot{W} = gQH_{pump}$

Internal flows



Low velocity

$Re = vD/\nu < \sim 2300$

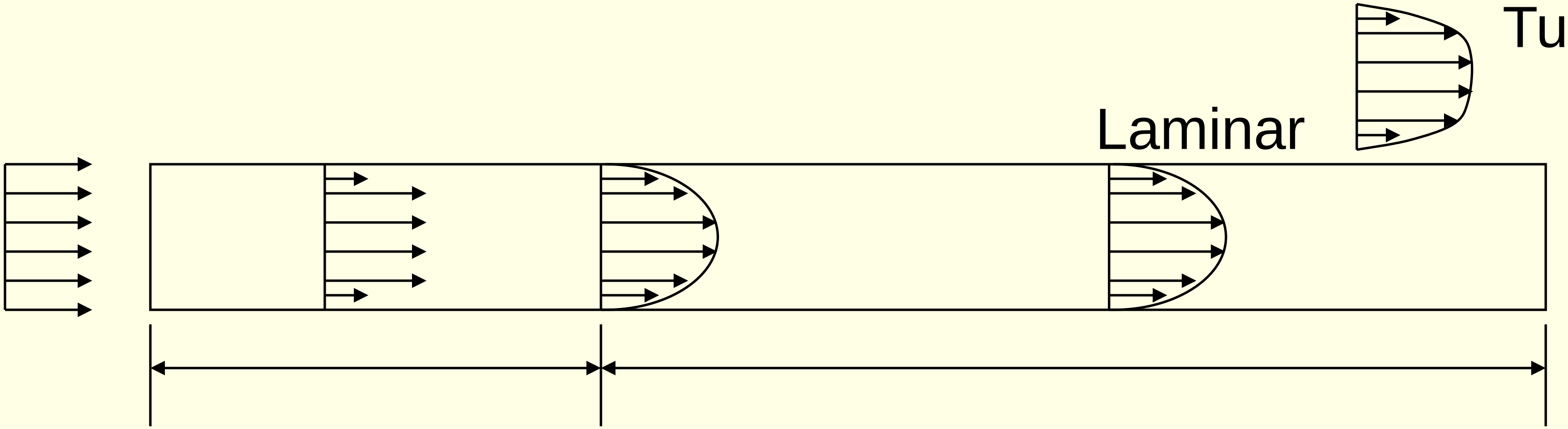
Laminar flow

High velocity

$Re = vD/\nu > \sim 2300$

Turbulent flow

Reynolds Experiment



Developing zone
L/D~20

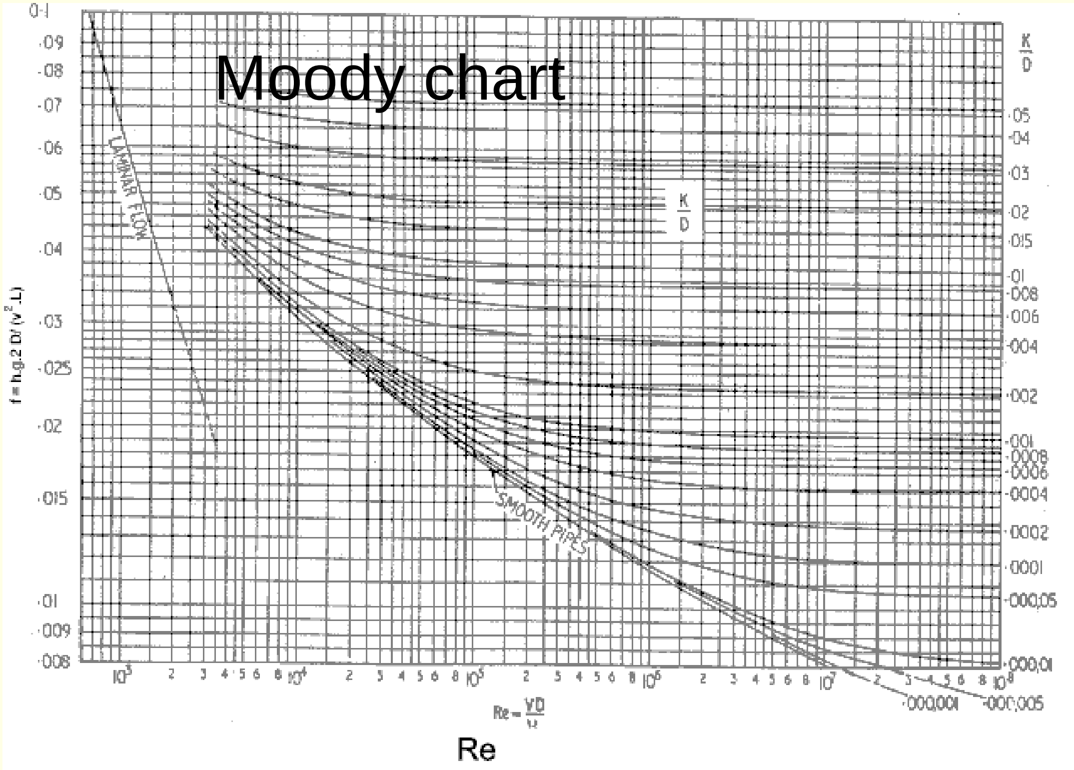
Fully developed zone

Losses in fully developed

$$\Delta P_{losses}/\rho = f\,L/D\,\mathrm{v}^2/2$$

$$h_{loss} = f\,L/D\,\mathrm{v}^2/2g$$

L = Pipe length
D = hydraulic diameter
= 4*Cross_Area/Perimeter
f = coefficient fn. of $Re=\mathrm{v}D/\nu$
and relative roughness K/D
For laminar flow: $f = Po/Re$
(Poiseuille no. $Po = 64$ for circular)
For turbulent flow: use chart



For minor losses
(elbows, valves, filters, …)

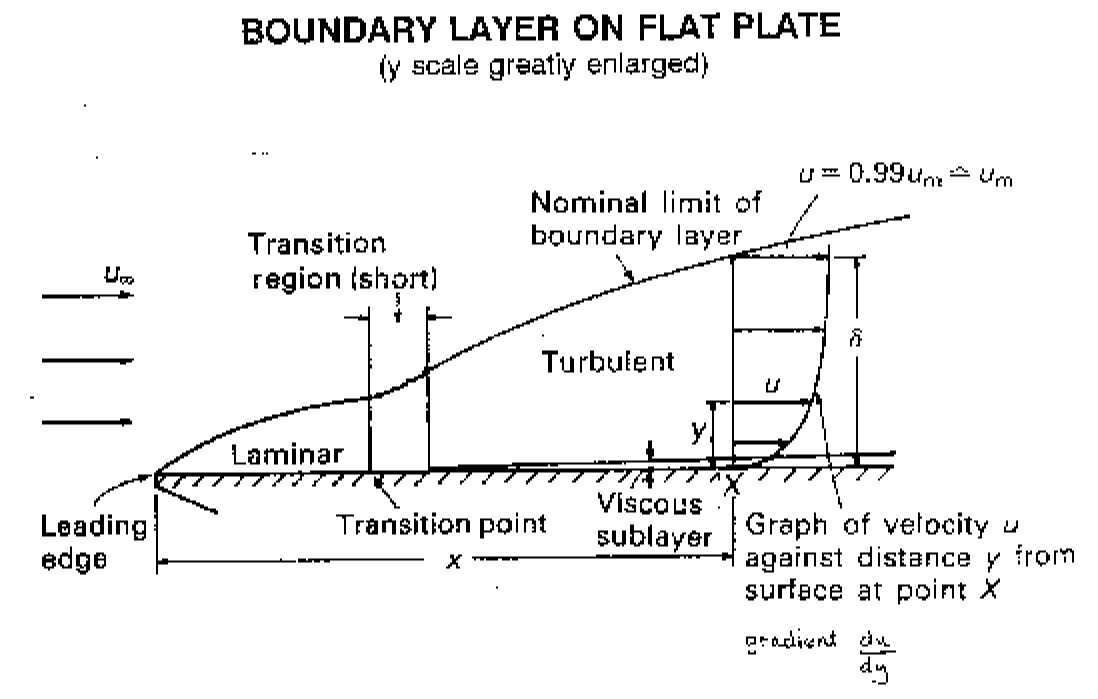
$$\Delta P_{losses}/\rho = k\,\mathrm{v}^2/2$$

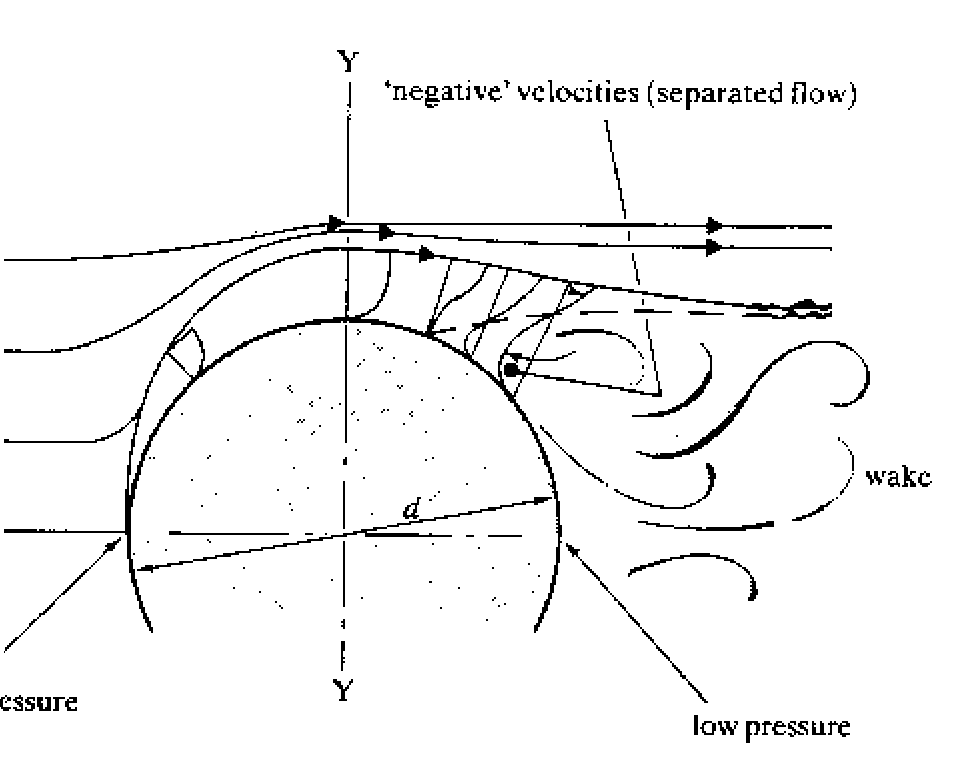
$$h_{losse} = k\,\mathrm{v}^2/2g$$

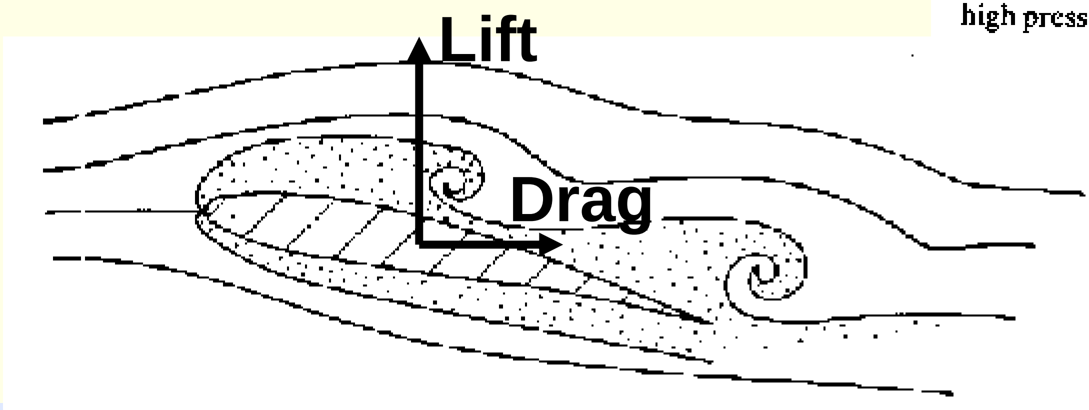
k is the loss coefficient obtained empirically. (See tables)

Additiona

External flow

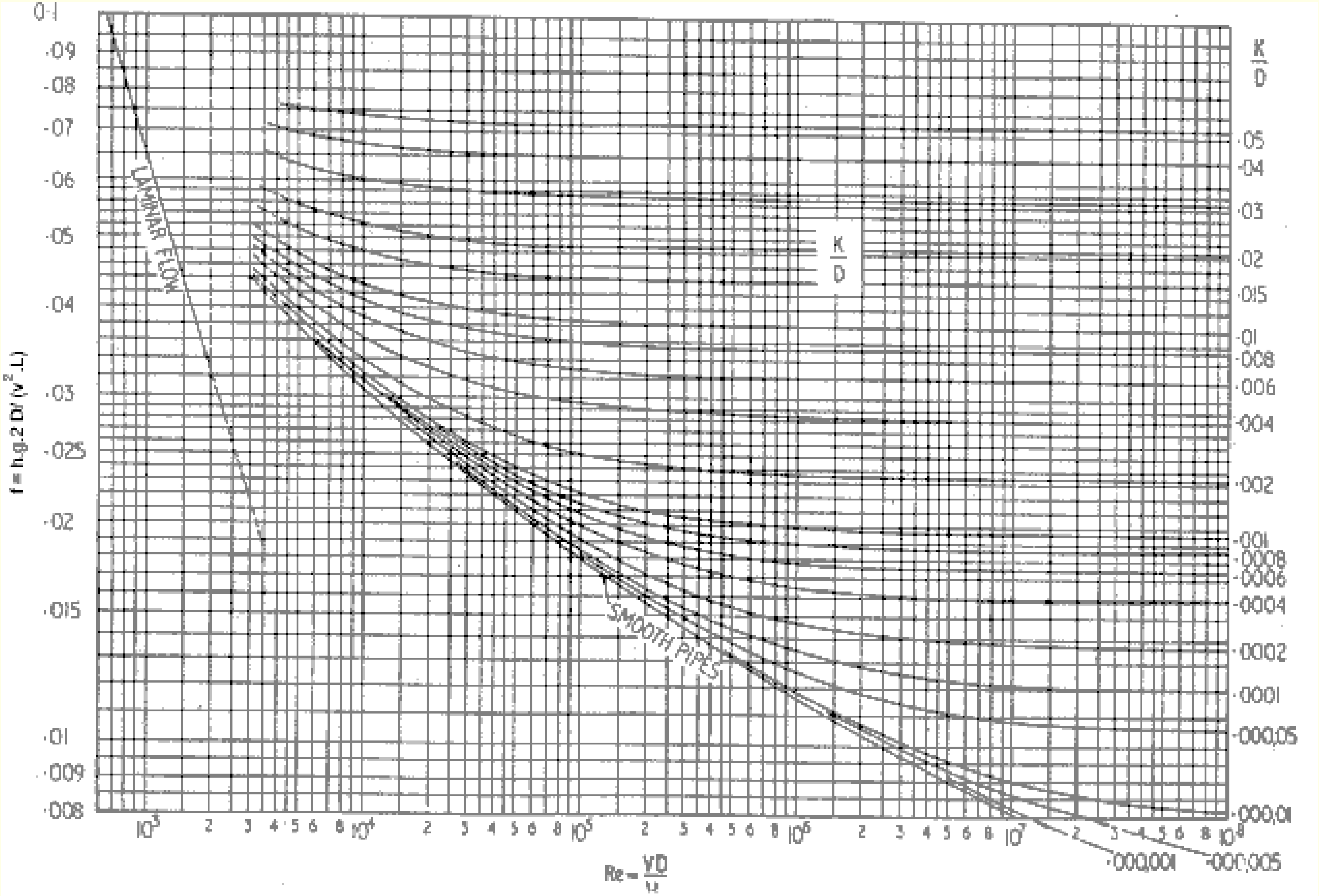






$$Lift = C_L\, A\, \rho\, v^2/2$$

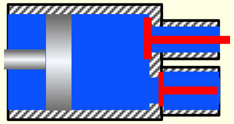
$$Drag = C_D\, A\, \rho\, v^2/2$$

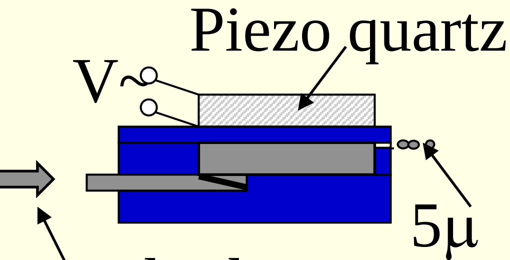

0·1
·09
·08
·07
·06
·05
·04
·03
·025
·02
·015
·01
·009
·008
f = h.g.2 D/ (v².L)
LAMINAR FLOW
K/D
SMOOTH PIPES
K/D
·05
·04
·03
·02
·015
·01
·008
·006
·004
·002
·001
·0008
·0006
·0004
·0002
·0001
·000,05
·000,01
·000,001
·000,005
10³
2
3
4
5
6
8
10⁴
2
3
4
5
6
8
10⁵
2
3
4
5
6
8
10⁶
2
3
4
5
6
8
10⁷
2
3
4
5
6
8
10⁸
Re = VD/ν

Re

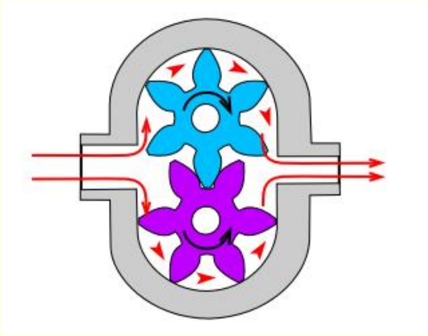
Classification of Fluid Machines

- Positive Displacement
- Turbo Machines
  - Work BY fluid
    - Impulse Turbine
    - Reaction Turbine
  - Work ON fluid
    - Liquid
      - Axial
      - Mixed
      - Radial
    - Gas
      - Fan
      - Blower
      - Compressor



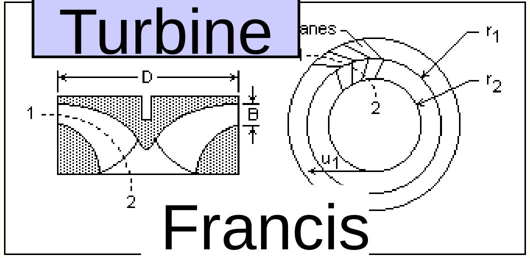


Ink Jet



Gear Pump

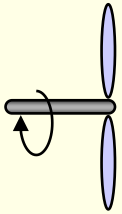
Pelton

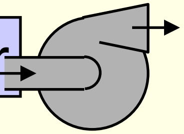


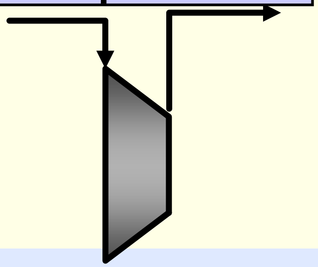
Francis

Low ΔP

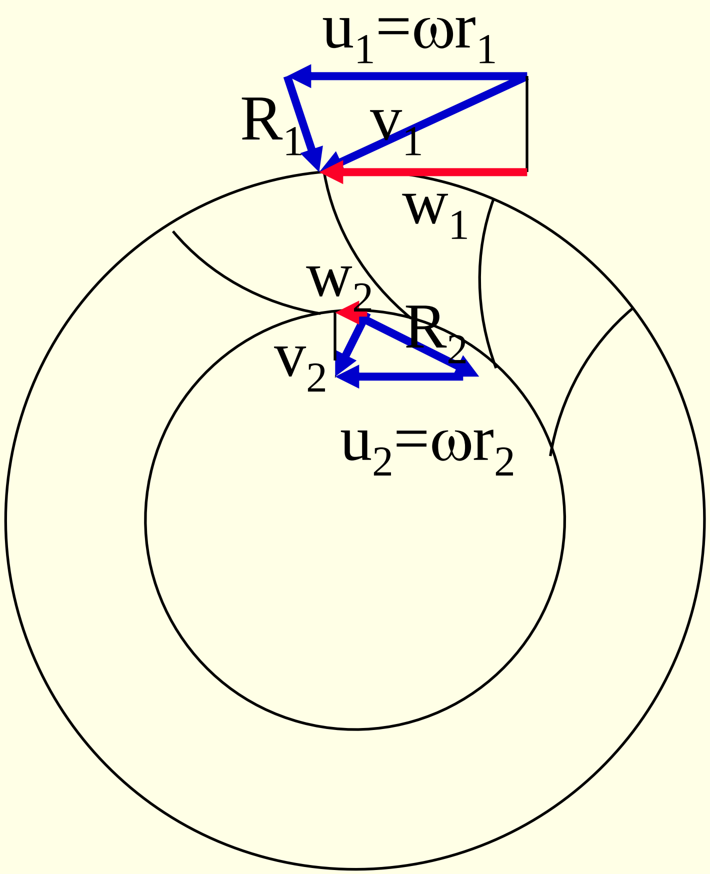
High ΔP







Fundamentals of Turbo Machines



u : wheel velocity = $\omega\, r \propto N\, D$

v : absolute velocity (N is the r.p.m.)

R : relative velocity

w : tangential component of v

Tangential momentum: $\rho\, Q\, .\, w$

Fluid Torque= $\rho\, Q\, .\, (w_1 r_1 - w_2 r_2)$

Fluid Power = $\rho\, Q\, .\, (w_1 r_1 - w_2 r_2)\, \omega$

$= \rho\, Q \quad (\Delta P/\rho)$ $\quad (\Delta P/\rho = gH)$

$\Rightarrow gH = w_1 u_1 - w_2 u_2$

Performance Description

For a given propeller design:

$u \propto ND$ (N speed, D outer diameter)

$$Q \propto u\ \text{Area} \Rightarrow Q/ND^3 \sim \text{Const } C_Q$$

$$gH \propto u^2 \Rightarrow gH/N^2D^2 \sim \text{Const } C_H$$

If N_s is defined as (N in rpm)

$$N_s = NQ^{0.5}/(H)^{0.75}$$

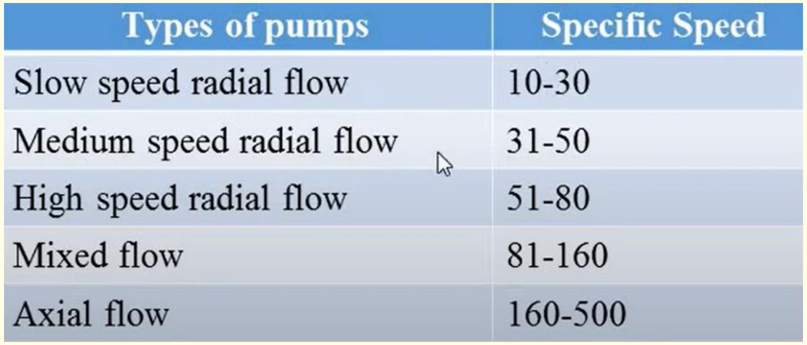
Q in l/s ; H in m

$\Rightarrow$

Specific speed in SI (N in rad/s)

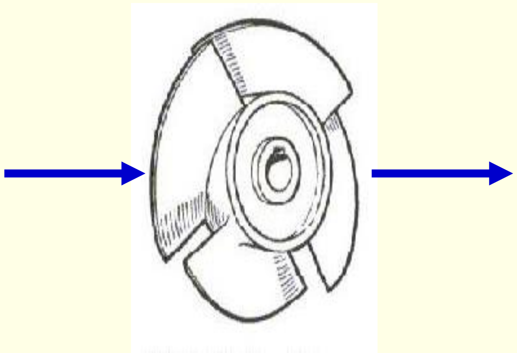
$$N_s = C_Q^{0.5}/C_H^{0.75}$$

$$= NQ^{0.5}/(gH)^{0.75}$$ D disappears!

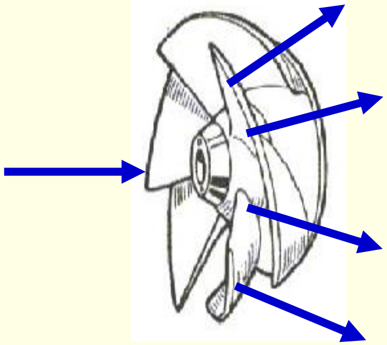


Types of pumps	Specific Speed
Slow speed radial flow	10-30
Medium speed radial flow	31-50
High speed radial flow	51-80
Mixed flow	81-160
Axial flow	160-500

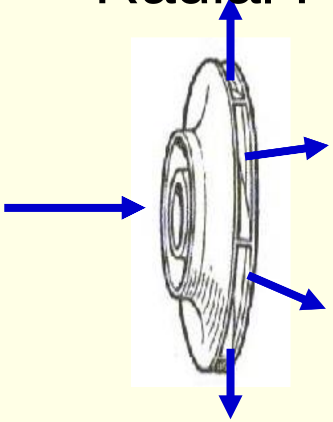
Axial Flow



Mixed Flow



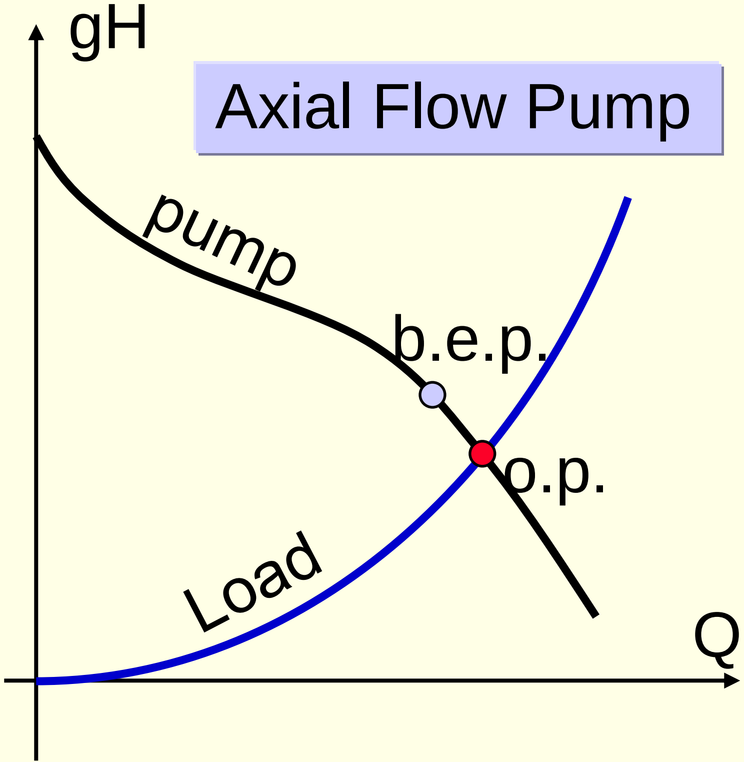
Radial Flow



High speed and/or Q; low ΔP

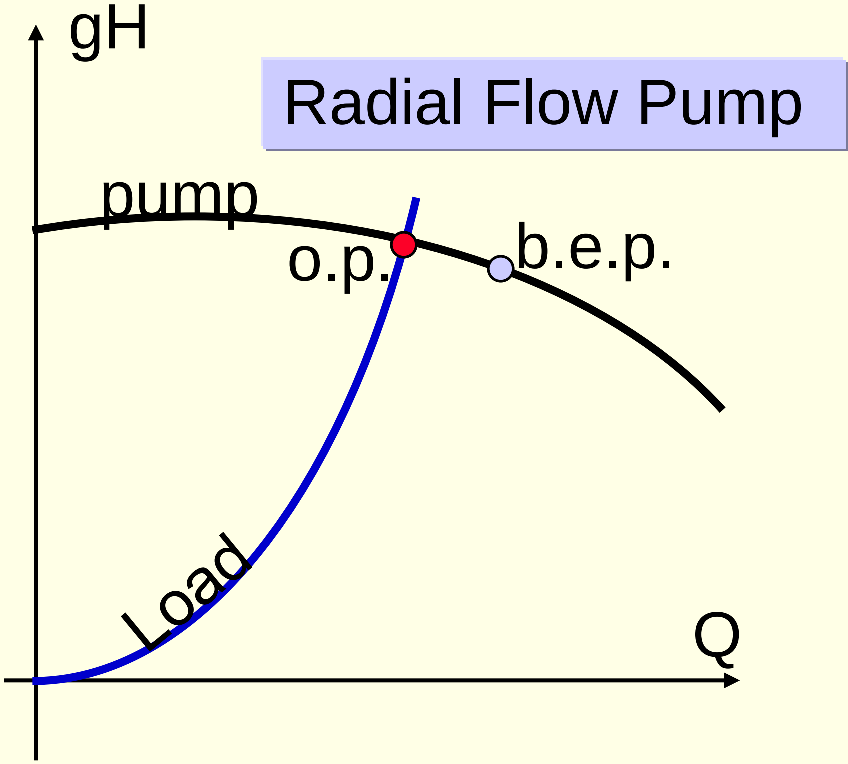
Low speed and/or Q; High ΔP

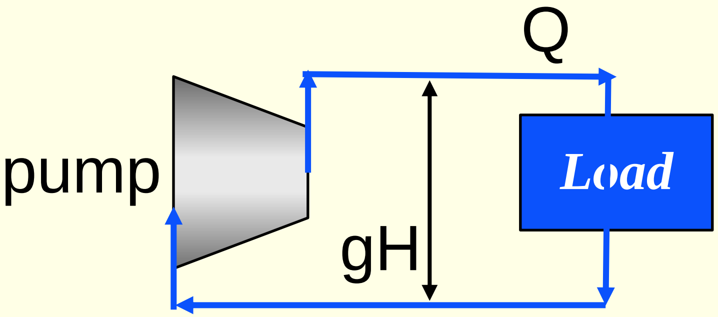
Typical Performance Curves



b.e.p.
=Best
Efficiency
Point

o.p.
=Operating
Point





To retain for chapter 9

Mass balance $0 + \dot{m}_{in} - \dot{m}_{out} = dm_s/dt$

$$\boxed{\dot{m} = \int \rho \mathbf{v} \cdot \mathbf{n} dA \approx \rho v_n A}$$

Momentum $\mathbf{F} + \dot{m}_{in}\mathbf{v}_{in} - \dot{m}_{out}\mathbf{v}_{out} = d(m_s\mathbf{v}_s)/dt$

Energy (1st Law)

For steady adiabatic flow of liquids: $q=0$ $w=\Delta P_{pump}/\rho$ $\Delta h=\Delta P/\rho + \Delta u$

Bernoulli's eq.:

$$\boxed{\Delta P_{pump}/\rho - \Delta u = \Delta P/\rho + \Delta v^2/2 + g\Delta z}$$

Losses due to friction

Reynolds number Re = v D_h/ν, v in m/s, D_h in m, ν in m^2/s
D_h = 4 Area / Perimeter

Pump power
$\dot{W}= \dot{m}\Delta P_{pump}/r$
$\dot{W}=g\dot{Q}H_{pump}$

Losses in ducts:

Major losses: $\Delta P_{losses}/\rho = f\, L/D_h\ v^2/2$

Minor losses: $\Delta P_{losses}/\rho = k\ v^2/2$

Friction coef f is f(Re, relative roughness) see Moody chart

Fluid machines types Positive displacement – Radial – Axial - Mixed

Fluid machines sizing Selecting by specific speed: Ns = N $\sqrt{Q}/(gH)^{3/4}$, N in rpm, Q in l/s, H in m
selecting adequate diameter and rotating speed

Operating point – Best efficiency point

Geothermal Energy Capacity Building in Egypt (GEB)

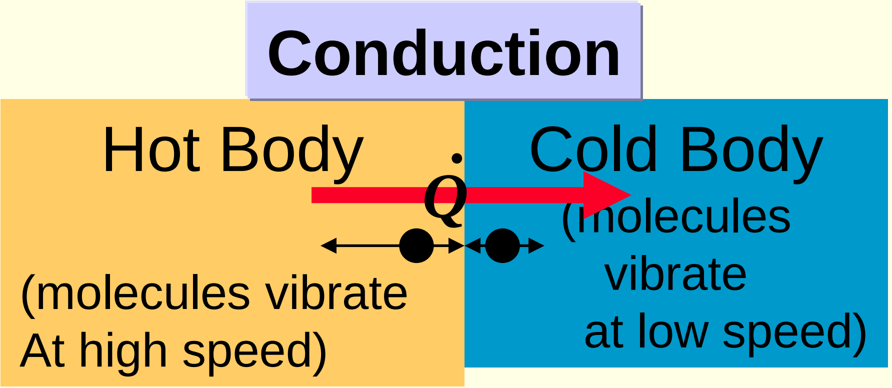
Thermal Engineering for Geothermal Energy

10 – Heat transfer

Co-funded by the
Erasmus+ Programme
of the European Union

Heat Transfer Modes

Heat can be transferred by 3 different modes



Conduction

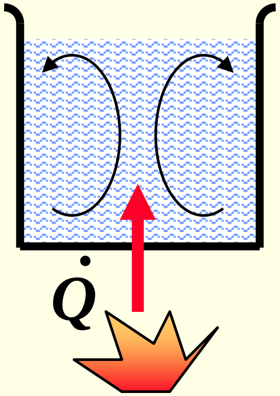
Hot Body

$\dot{Q}$

Cold Body

(molecules vibrate
At high speed)

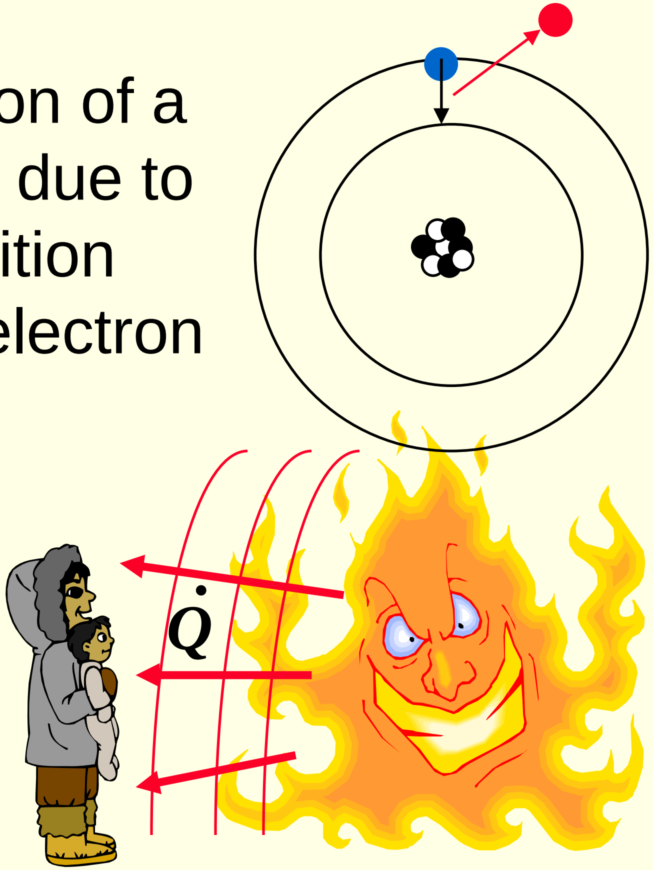
(molecules
vibrate
at low speed)



Convection

$\dot{Q}$

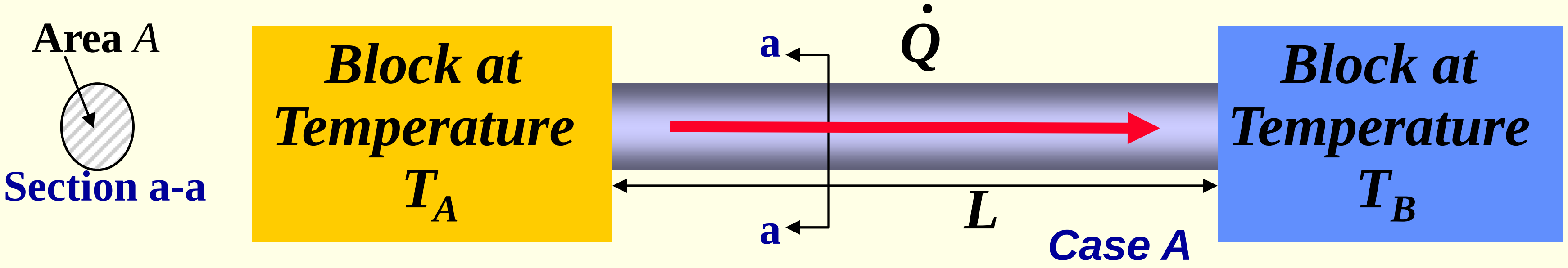
Emission of a
photon due to
a transition
Of an electron



$\dot{Q}$

Radiation

Conduction mono – dimensional 1



Area A

Section a-a

Block at Temperature T_A

a

$\dot{Q}$

a

L

Block at Temperature T_B

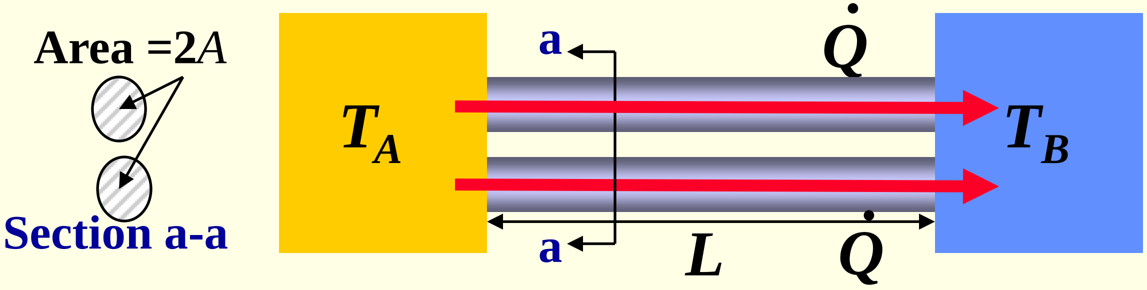
Case A

$\dot{Q}$ *depends on what***?**

- **On the material**
- **on $\Delta T = T_A - T_B$**
- **On dimensions L & A**

To find the relation,
First remark:

- **if 2 bars identical & parallel,**
  Area A doubles, $\dot{Q}$ doubles, $\Rightarrow \dot{Q} \propto A$



Area =2A

Section a-a

T_A

a

$\dot{Q}$

a

L

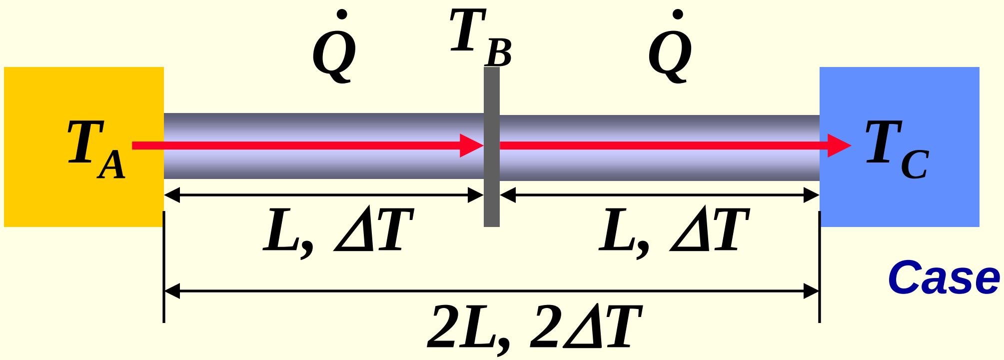
$\dot{Q}$

T_B

Case B

$$\dot{q} = \dot{Q}/A = f\left(\Delta T, L, mat.\right)$$

Conduction mono – dimensional 2



Case C

If 2 bars identical in series,
With $\Delta T = T_A - T_B = T_B - T_C$
$\Rightarrow$ Same $\dot{q}$ in each bar
i.e. If L doubles & ΔT doubles:
$\dot{q}$ does not change
$\Rightarrow \dot{q} = f(\Delta T / L, \mathit{matter})$

$$\dot{q} = -k\,(T_B - T_A)/L$$

Limit for $L = dx \rightarrow 0$*:*

$$\boxed{\dot{q} = -k\, dT/dx}$$

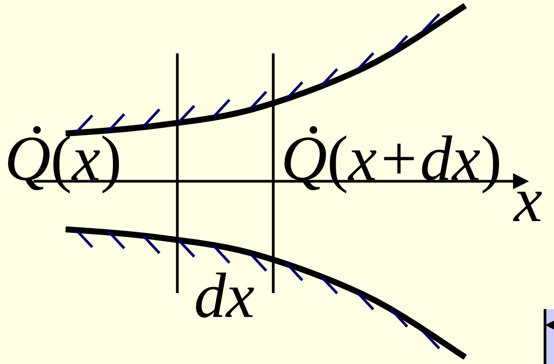
Fourier Law

NB:
- **If** $\Delta T/L = 0$ **then** $q \doteq 0$

$$\dot{q} = f\left(\Delta T/L, mat.\right)$$
$$\cong k\left(T_A - T_B\right)/L$$

Thermal Conductivity

Conduction in a variable section

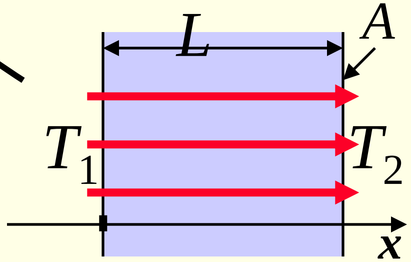


$$\dot{Q}(x) = -\,kAdT/dx \qquad (1)$$

$$d\dot{Q}(x)/dx = 0 = d(kAdT/dx)/dx \qquad (2)$$

$$d\dot{Q}(x)/dx = q_vA = -d(kAdT/dx)/dx \qquad (3)$$

Case of a block:
(q_v=0)

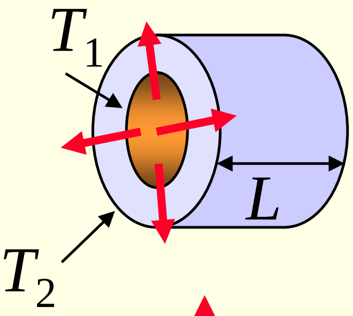


(2) ⇒ $kAd^2T/dx^2 = 0;$

i.e. $T=T_1+(T_2-T_1)x/L$

(1) ⇒ $\dot{Q}=kA/L\,(T_1-T_2)$

Case of a cylinder:
(q_v=0)

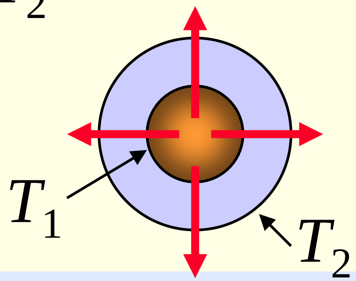


(2) ⇒ $k\,2\pi L\,d(r\,dT/dr)/dr = 0$

i.e. $T=T_1+(T_2-T_1)\,ln(r/r_1)/ln(r_2/r_1)$

(1) ⇒ $\dot{Q}=2\pi kL\,(T_1-T_2)/\,ln(r_2/r_1)$

Case of a sphere:
(q_v=0)

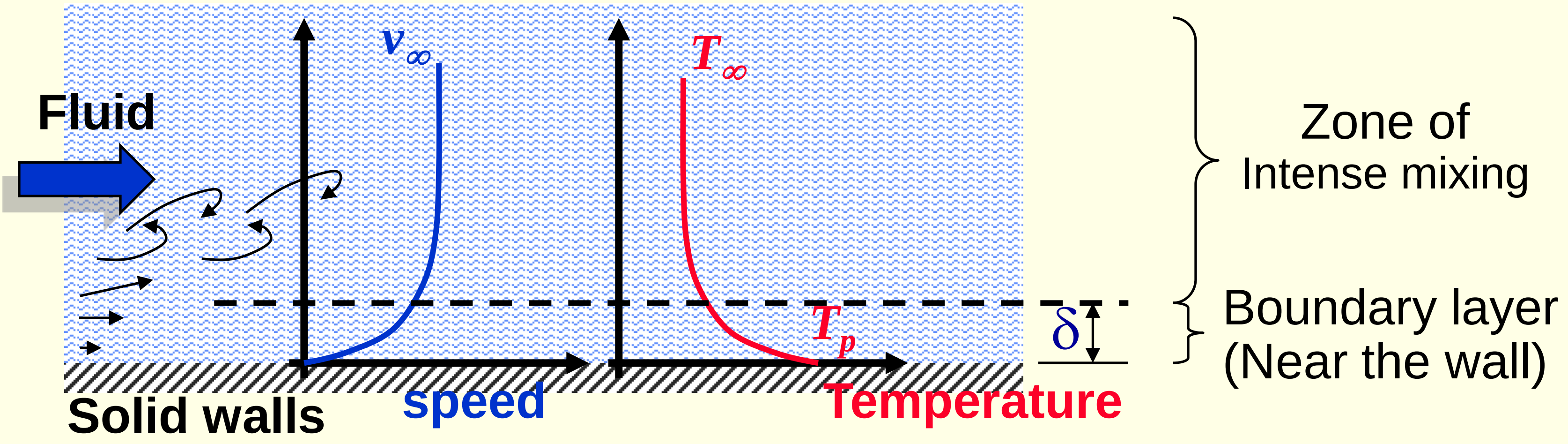


(2) ⇒ $k\,4\pi\,d(r^2\,dT/dr)/dr = 0$

i.e. $T=T_1+(T_2-T_1)\,(1-r_1/r)/(1-r_1/r_2)$

(1) ⇒ $\dot{Q}= 4\pi k\,(T_1-T_2)r_1/(1-r_1/r_2)$

Transfer by convection

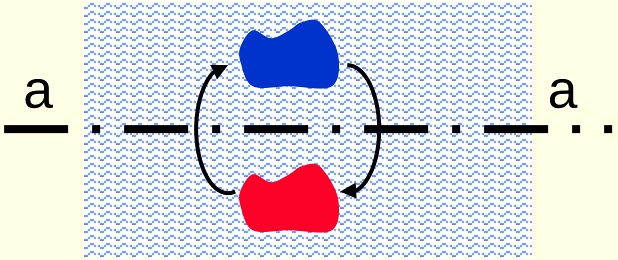


In the boundary layer (Low velocity): $\dot{q} \approx (k/\delta)\,(T_\mathrm{p} - T_\infty)$

Case of « turbulent » exchange:

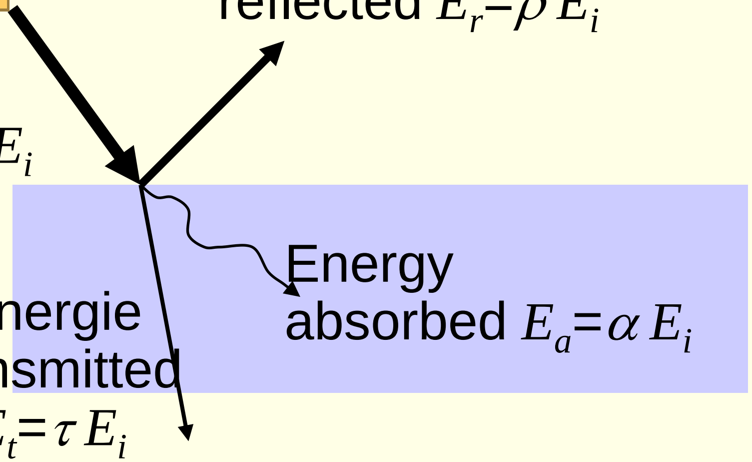
Due to eddies,
an exchange of packets of fluid
(hot/cold) takes place through surface a-a

➢Associated heat: $\dot{q} \propto (T_A - T_B)$



In general: $\boxed{\dot{q} = h\,\Delta T}$ Newton's law of cooling

Transfer by radiation: 1- absorption



Incident Ray

Energy reflected $E_r = \rho E_i$

Energy incident E_i

Energy absorbed $E_a = \alpha E_i$

Energie transmitted $E_t = \tau E_i$

By conservation:
$\alpha + \rho + \tau = 1$

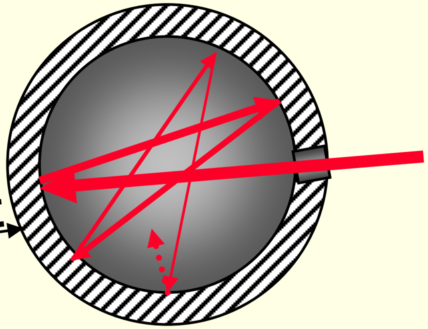
Special cases

Perfect mirror: $\rho = 1$

Perfect transparent : $\tau = 1$

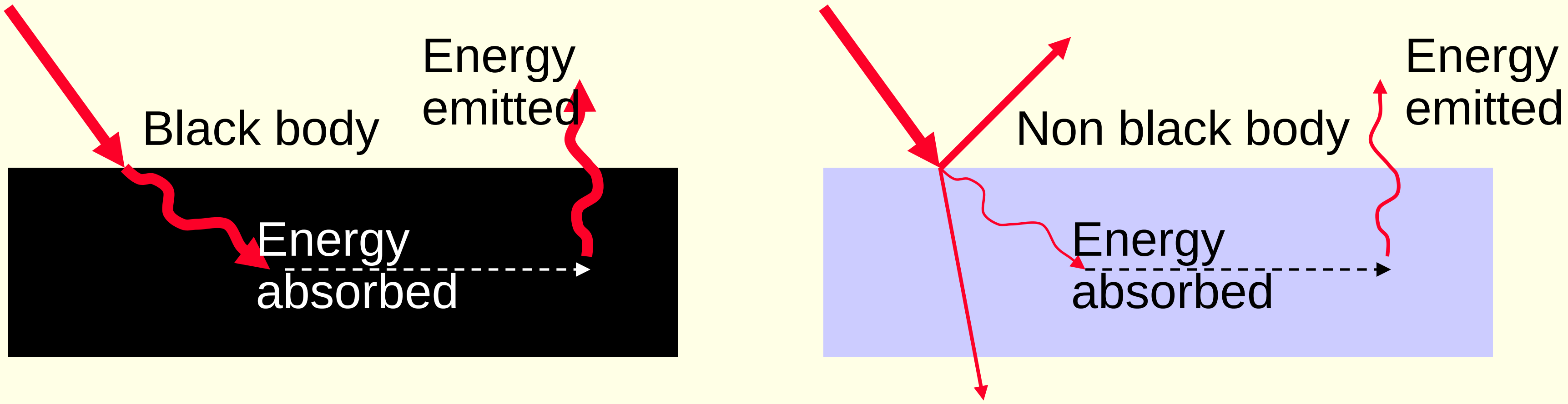
Perfect absorbent
= **Black body**: $\alpha = 1$

Example:



2- Emission

Every body, at any temperature, emits thermal radiation



Black body

Energy emitted

Energy absorbed

Non black body

Energy emitted

Energy absorbed

In case of equilibrium, energy absorbed = energy emitted

At the same temperature,

Black body (b.b.) emits the most: $\dot{q}_{bb} = \sigma T^4$

$\sigma = 5.67*10^{-8}\ \mathrm{W/K^4}$

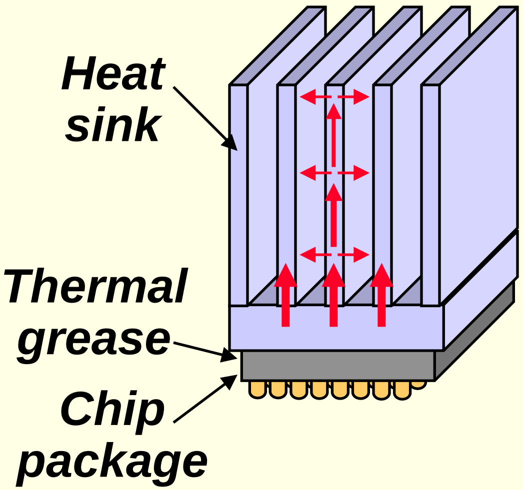
Coefficient of Stefan - Boltzmann

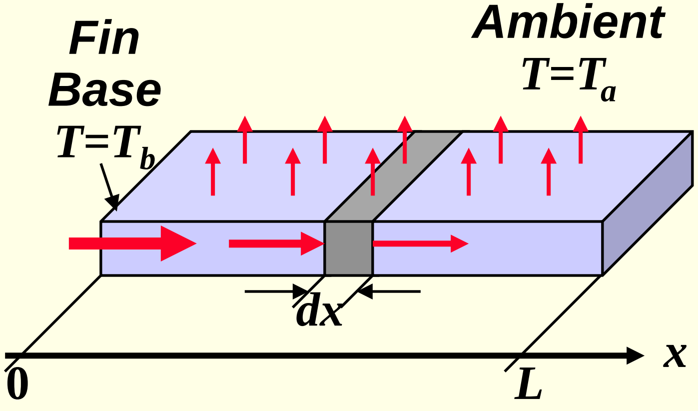
In general, $\dot{q} = \varepsilon \sigma T^4$ where ε = emissivity ≤ 1

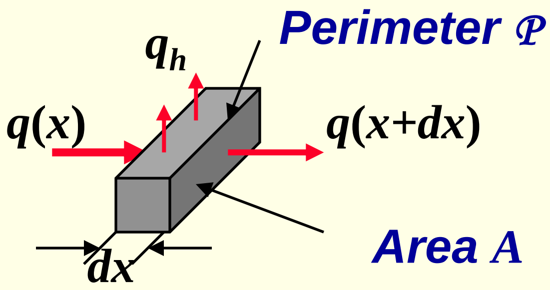
Kirchhoff Law: in case of equilibrium (same temperature) $\boxed{\varepsilon = \alpha}$

Additional

Fins







$$[q(x+dx) - q(x)]\,A = q_h\,\mathcal{P}\,dx$$

$$q(x) = -k\,dT/dx$$

$$q_h = h\,(T-T_a)$$

$$\boxed{d^2T/dx^2 - m^2(T-T_a) = 0}$$

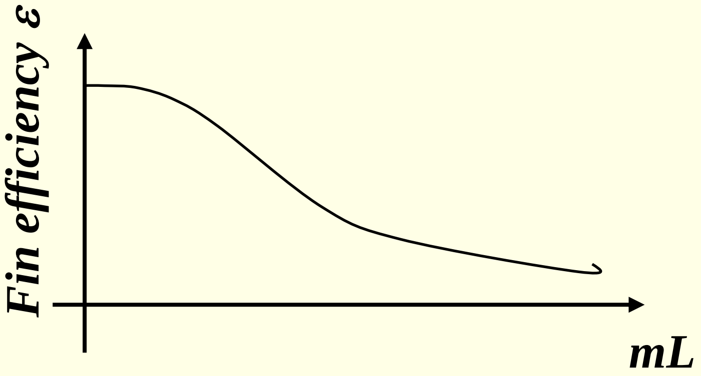
$$\boxed{m^2 = h\mathcal{P}/kA}$$

$$\boxed{\text{If } T(0)=T_b;\ dT/dx\,|_{x=L} = 0}$$

$$\Rightarrow\ T-T_a = (T_b - T_a)\cosh(m(L-x))/\cosh(mL)$$

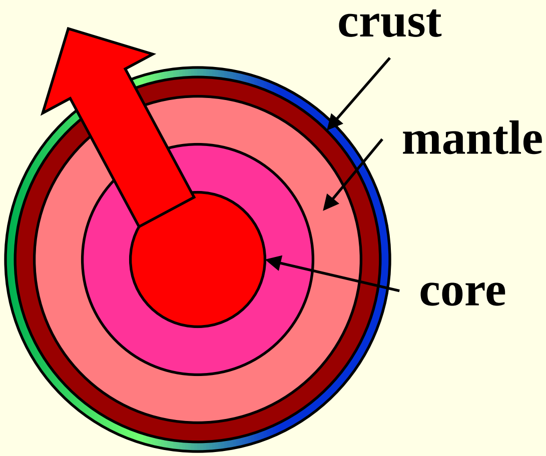
$$Q = -kA\,dT/dx|_{x=0} = kAm(T_b-T_a)\tanh(mL)$$

$$\text{Fin efficiency } \varepsilon = Q/Q\max = Q/(hL\mathcal{P}(T_b-T_a)) = \tanh(mL)/mL$$



Geothermal temperature gradient

According to the
International Union of Geodesy and Geophysics



Geothermal out power = 44.2 TW

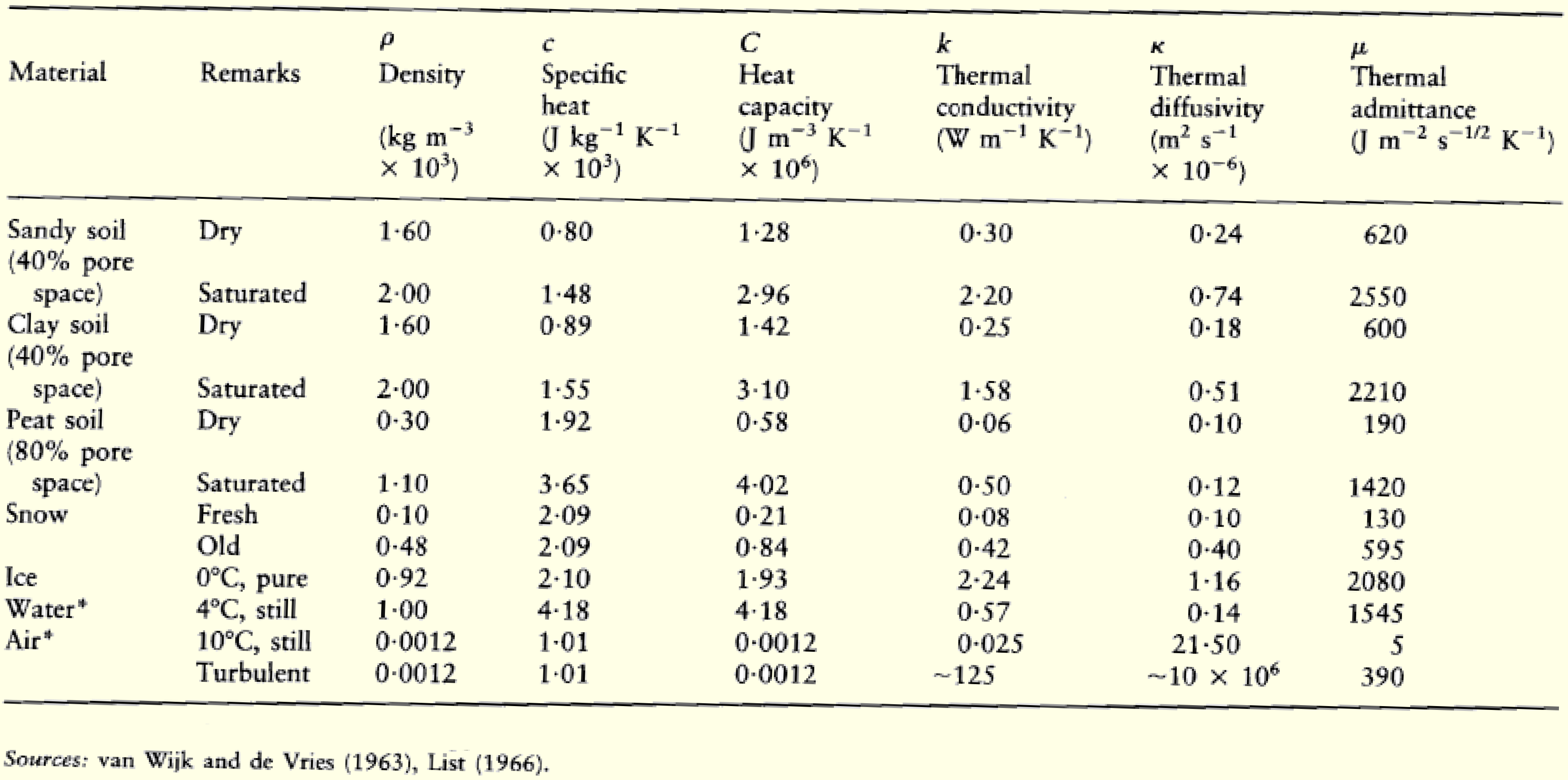
> 2 times world consumption

BUT low density: 0.087 W/m^2

Replenished by 30 TW of radioactive decay

Geothermal thermal gradient in earth crust ~ 25 – 30^{o}C/km

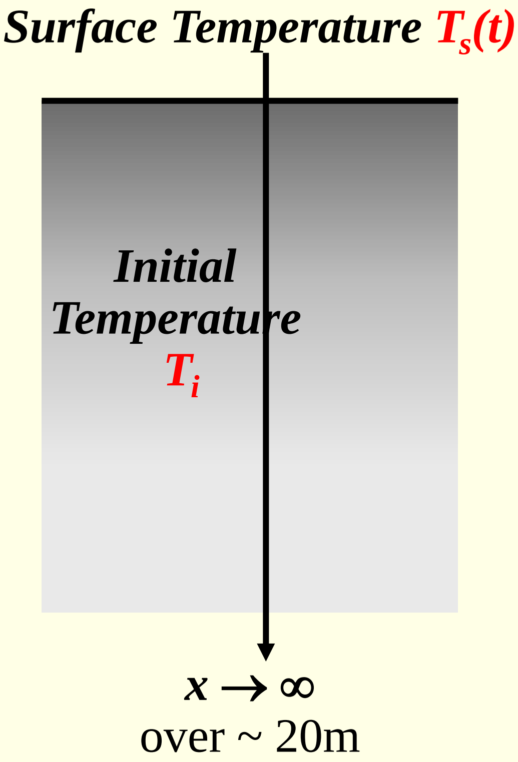
Soil heat transfer properties



Material	Remarks	ρ Density ($kg\ m^{-3} \times 10^3$)	c Specific heat ($J\ kg^{-1}\ K^{-1} \times 10^3$)	C Heat capacity ($J\ m^{-3}\ K^{-1} \times 10^6$)	k Thermal conductivity ($W\ m^{-1}\ K^{-1}$)	κ Thermal diffusivity ($m^2\ s^{-1} \times 10^{-6}$)	μ Thermal admittance ($J\ m^{-2}\ s^{-1/2}\ K^{-1}$)
Sandy soil (40% pore space)	Dry	1·60	0·80	1·28	0·30	0·24	620
	Saturated	2·00	1·48	2·96	2·20	0·74	2550
Clay soil (40% pore space)	Dry	1·60	0·89	1·42	0·25	0·18	600
	Saturated	2·00	1·55	3·10	1·58	0·51	2210
Peat soil (80% pore space)	Dry	0·30	1·92	0·58	0·06	0·10	190
	Saturated	1·10	3·65	4·02	0·50	0·12	1420
Snow	Fresh	0·10	2·09	0·21	0·08	0·10	130
	Old	0·48	2·09	0·84	0·42	0·40	595
Ice	0°C, pure	0·92	2·10	1·93	2·24	1·16	2080
Water*	4°C, still	1·00	4·18	4·18	0·57	0·14	1545
Air*	10°C, still	0·0012	1·01	0·0012	0·025	21·50	5
	Turbulent	0·0012	1·01	0·0012	~125	$\sim 10 \times 10^6$	390

Sources: van Wijk and de Vries (1963), List (1966).

Transient conduction in near soil



Governing equation:

$$\boxed{\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2}}$$

Thermal diffusivity α in m^2/s

Initial condition: $T(x,0) = T_i$

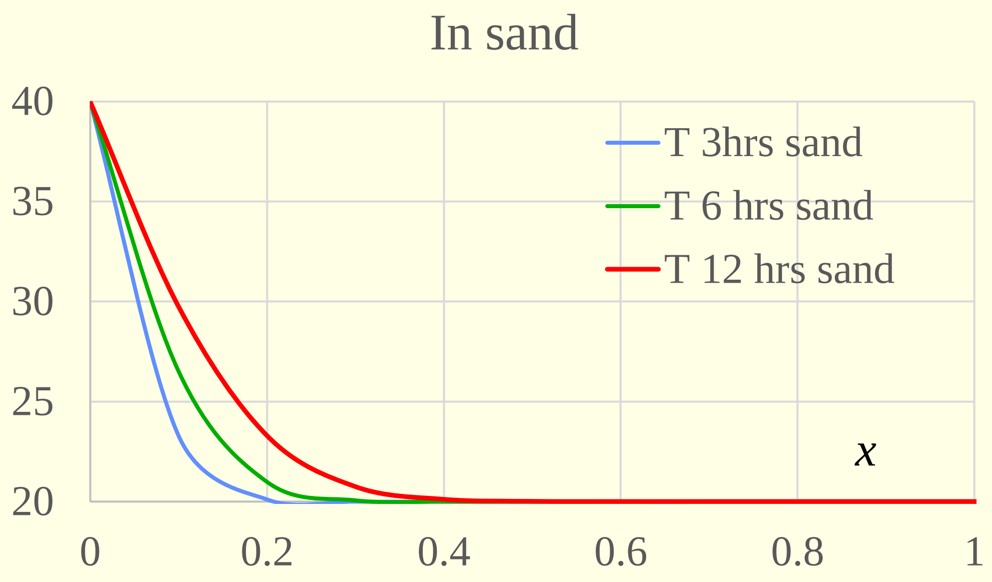
Boundary conditions:

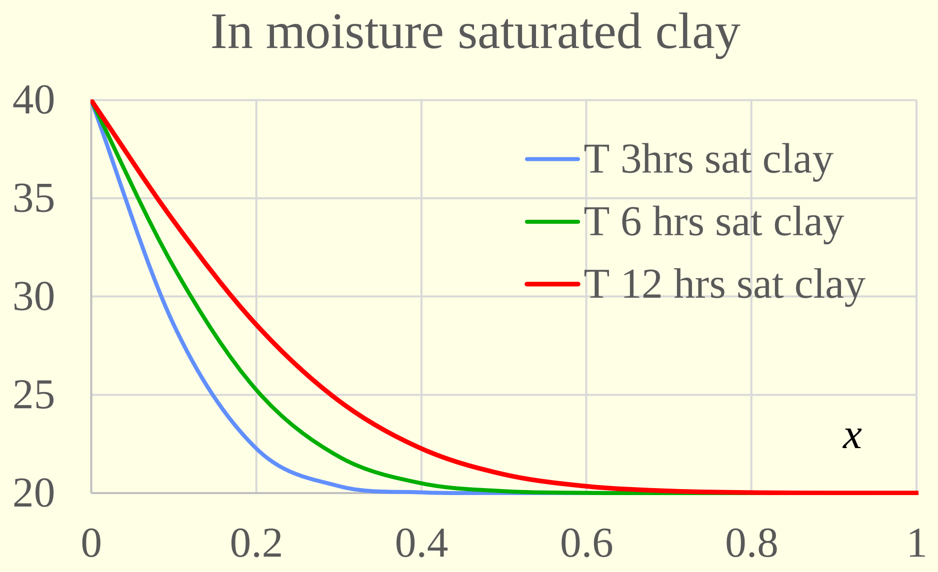
$$T(x \to \infty, t) = T_i$$

$$T(x = 0, t) = T_s(t)$$

Effect of sudden surface T change

Assuming: $T_i = 20^oC$, $T_s = 40^oC$
$\alpha_{sand} = 2.4\ 10^{-6}\ m^2/s$, $\alpha_{sat\ clay} = 7.4\ 10^{-6}\ m^2/s$

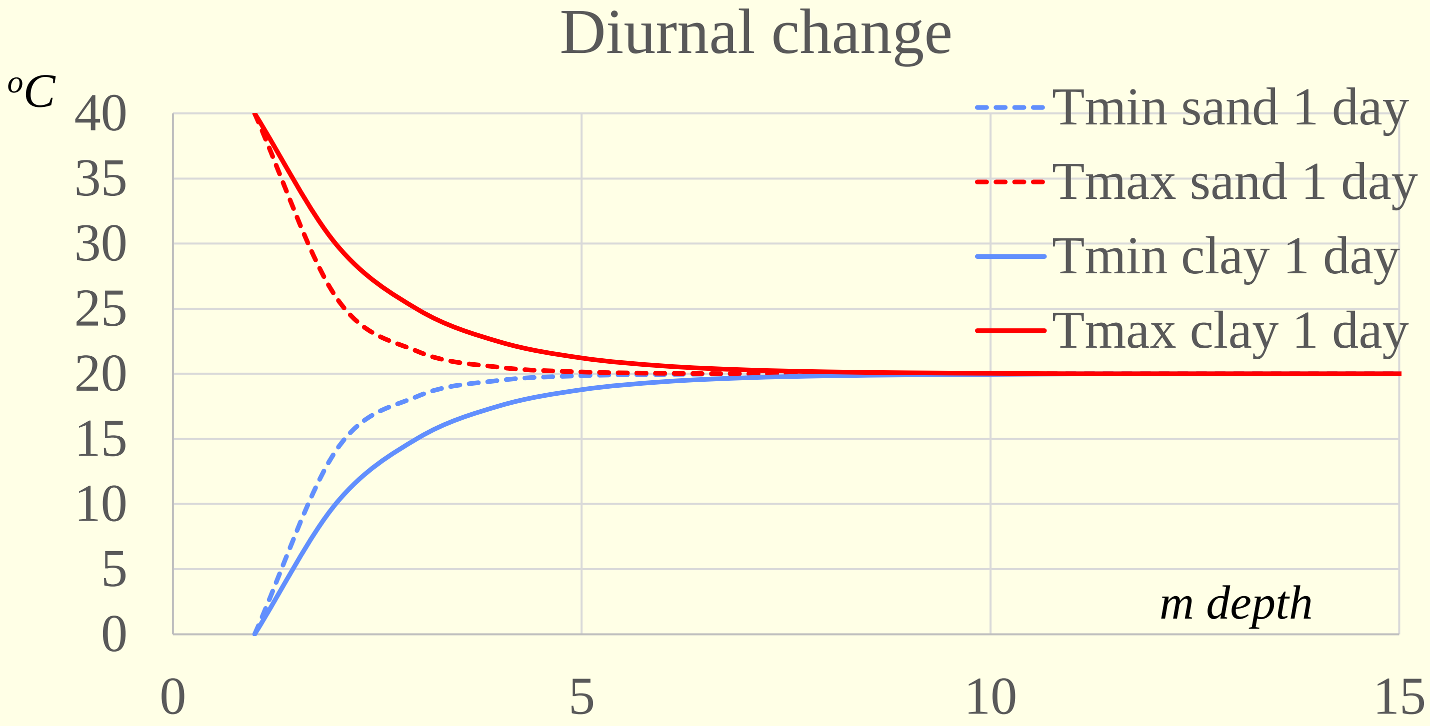




Effect of diurnal variations

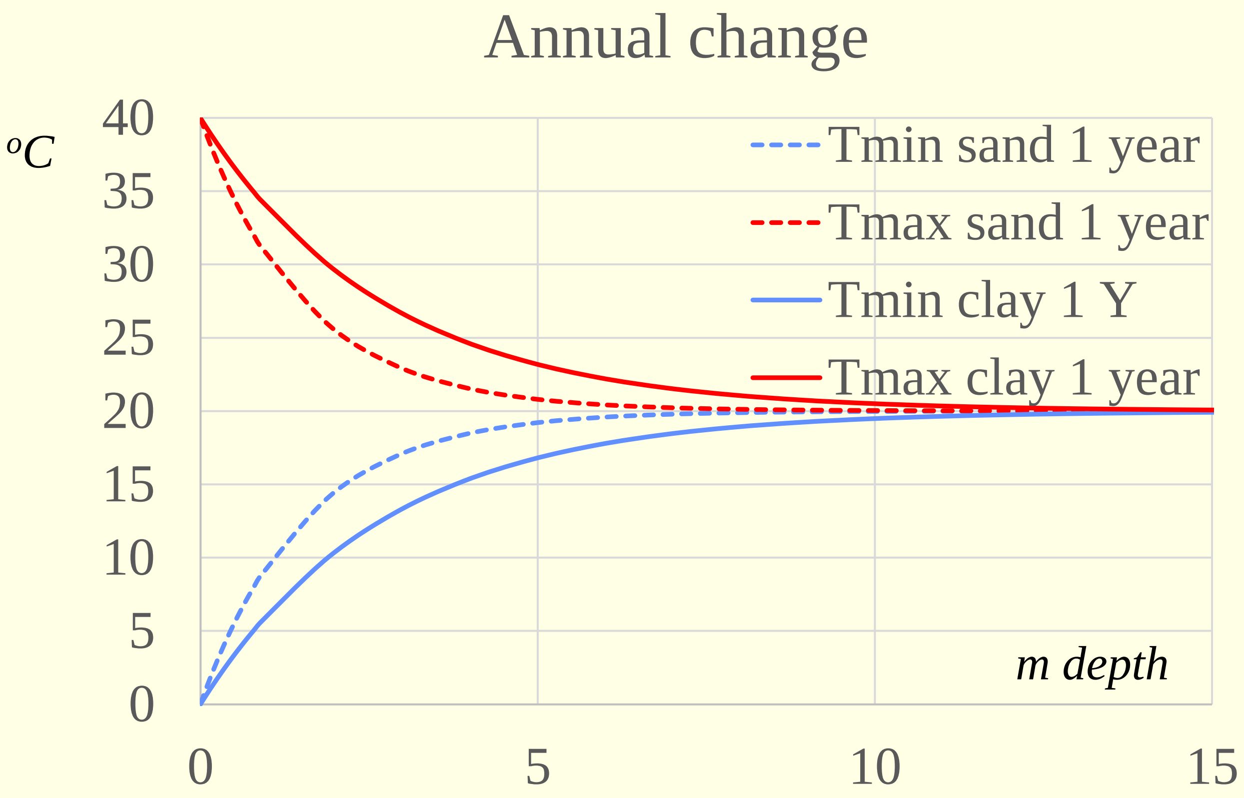
Assuming diurnal swing is: $T_{min} = 0^oC$, $T_{max} = 40^oC$

$\alpha_{sand} = 2.4\ 10^{-6}\ m^2/s$, $\alpha_{sat\ clay} = 7.4\ 10^{-6}\ m^2/s$



Effect of annual variations

Assuming annual swing is: T_{min} = 0°C, T_{max} = 40°C
$\alpha_{sand} = 2.4\ 10^{-6}\ m^2/s$, $\alpha_{sat\ clay} = 7.4\ 10^{-6}\ m^2/s$



To retain for chapter 10

Heat transfer modes *Conduction – Convection – Radiation*

Steady conduction ***Fourier Law*** $q = -k\, dT/dx$

For a block $T=T_1+(T_2-T_1)x/L \quad Q=kA/L\,(T_1-T_2)$

For a cylinder $T=T_1+(T_2-T_1)\, ln(r/r_1)/ln(r_2/r_1) \quad Q=2\pi\, kL\,(T_1-T_2)/\, ln(r_2/r_1)$

For a sphere $T=T_1+(T_2-T_1)\,(1-r_1/r)/(1-r_1/r_2) \quad Q=4\pi k\,(T_1-T_2)r_1/(1-r_1/r_2)$

Convection $q = h\,\Delta T$

Radiation $q = \varepsilon\,\sigma\, T^4$ $\sigma = 5.67*10^{-8}$ W/K^4

Applications to geothermal **Geothermal thermal gradient in earth crust ~ 25 – 30°C/km**

Transient conduction **Temperature swing near earth surface: disappears in ~ 10m**